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SOCIALLY RESPONSIBLE INVESTMENT-BASED PORTFOLIO SELECTION PROBLEMS WITH FUZZINESS

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Abstract. *This paper considers several portfolio selection problems considering Socially Responsible Investment (SRI), which is the most important measure to sustain continuous developments of companies by performing environment-friendliness and suitable social activity, and which is also essential for avoiding the latent risk. Corporate Social Responsibility (CSR) is presented as linguistic and ambiguous information including several types of subjectivities, and so effects of SRI activities for the investment based on the CSR are formulated as fuzzy numbers. Furthermore, several types of portfolio models, considering direct evaluation approach of SRI, fuzzy variance including SRI, and biased future return derived from random simulation, are proposed. In order to evaluate these portfolio performances, a practical example derived from the current market is provided.*
Keywords: Portfolio selection problem, Socially responsible investment, Fuzzy theory

1. **Introduction.** A portfolio selection problem has been one of standard and most important problems in investment and financial research fields since the mean-variance model proposed by Markowitz [11]. It has been central to research activities in the real financial field and numerous researchers have contributed to the development of modern portfolio theory (for instance, [2]), and many researchers have proposed several types of portfolio models to extend Markowitz model, mean-absolute deviation model [8, 9], safety-first model [2], Value-at-Risk and conditional Value-at-Risk model [12], etc. As a result, nowadays it is common practice to extend these classical economic models of financial investment to various types of portfolio models because investors correspond to present complex markets. In practice, many researchers have been trying different mathematical approaches to develop the theory of portfolio model.

Investors receive effective or ineffective investment and financial information from the real market, which usually contain ambiguous factors. Even if investors hold sufficient information from the investment field, it is still difficult to predict each asset's present or future probability distribution due to other uncertainties, such as the investor's subjectivity. Consequently, we need to consider not only random conditions but also other ambiguous conditions for portfolio selection problems. In recent studies on mathematical programming, certain researchers have proposed various types of portfolio models under randomness and fuzziness. These problems with probabilities and possibilities are generally called stochastic programming problems and fuzzy programming problems, respectively. Certain basic studies use a stochastic programming approach and goal programming approach for randomness, as well as fuzzy programming approach to treat ambiguous factors as fuzzy sets (for instance, [3-7, 10, 14-17]).

In contrast, over the past few years, many investors have begun to take into account a new factor, Corporate Social Responsibility (CSR) and Social Responsibility Investment (SRI), for selecting their investments. CSR is currently the most important measure for companies' sustained continuous development through environmental friendliness and suitable social activity. Moreover, it is essential for avoiding latent risk. CSR is not just a prominent research theme; it can also be found in corporate missions and value statements for long-term success [1, 13]. Furthermore, SRI is an investment strategy which integrates social or environmental criteria into financial analysis. That is, it is an investment approach not only focusing on companies concerned with ethical issues but also avoiding investing in companies which produce certain products, such as armaments, or follow certain policies, such as discrimination against minorities. Thus, the field of socially responsible mutual funds has become an area of growing interest within the modern financial sphere, considering not only their return and risk but also their social responsibility profile. Therefore, in order to evaluate SRI performance, we need a social responsibility measure which can be used as an output variable to be considered together with the return and risk measures. Most recently, definitions of SRI measures have proliferated in related research. Thus, we consider several types of portfolio models for SRI evaluation.

CSR and SRI are usually presented as linguistic and ambiguous information including several types of subjectivities. Hence, it is hard to consider these measures as fixed values. In this study, in terms of linguistic properties and subjectivity, we formulate the effects of SRI activities for investments as fuzzy numbers, and hence our proposed model is formulated as an uncertainty programming problem. By performing deterministic equivalent transformations and considering the application in practice, we develop an analytical and effective solution algorithm.

This paper is organized in the following way. In Section 2, we introduce mathematical formulations of standard portfolio models. In Section 3, we propose several types of portfolio models based on SRI. In this paper, we focus on the text-based aspect and negative screening; we introduce fuzzy numbers and fuzzy goals for negative evaluation values. In Section 4, in order to consider portfolio performances, we provide a numerical example. Finally, in Section 5, we conclude this paper.

2. Formulation of Standard Portfolio Models.

First, we introduce one of standard mathematical approaches for portfolio selection problems, mean-variance model called Markowitz model. Markowitz [11] proposed the following mathematical model as a portfolio selection problem:

$$\begin{array}{ll} \text{Minimize} & \boldsymbol{x}^t \mathbf{V} \boldsymbol{x} \\ \text{subject to} & \bar{\boldsymbol{r}}^t \boldsymbol{x} \geq r_G, \\ & \sum_{j=1}^{n} a_j x_j \leq b, \quad \boldsymbol{x} \geq \mathbf{0} \end{array} \tag{1}$$

where the notation of parameters is as follows:

$\bar{\boldsymbol{r}}$: Mean value of n-dimensional Gaussian random variable row vectors,
$\mathbf{V}$: Variance-covariance matrix,
r_G: Minimum value of the goal for expected total return,
a_j: Cost coefficient of jth asset,
b: Maximum value of total budget,
$\boldsymbol{x}$: Purchasing volume (an n-dimensional decision variable column vector).

This formulation has long served as the basis of financial theory. This problem is a quadratic programming problem, and so we find the optimal portfolio using standard convex and nonlinear programming approaches. However, it is not efficient to solve the large scale quadratic programming problem directly. Furthermore, in the case that the investor expects the future return of each product, she or he does not consider only one scenario of the future return, but often several scenarios.

In this regard, using many scenarios of future returns, the mean-variance model can be reformulated. Let r_{ij} be the realization of random variable R_j about the scenario i, $(i = 1, 2, \ldots, m)$, which we assume to be available from historical data and from investor's subjective prediction. Then, the return vector of scenario i is as follows:

$$\boldsymbol{r}_i = (r_{i1}, r_{i2}, \cdots, r_{in}), \quad i = 1, 2, \ldots, m \tag{2}$$

where n is the number of total asset. We introduce a probability for each scenario as follows:

$$p_i = \Pr\{\boldsymbol{r} = \boldsymbol{r}_i\}, \quad i = 1, 2, \ldots, m \tag{3}$$

We also assume that the expected value of the random variable can be approximated by the average derived from these data. Particularly, let the arithmetic mean

$$\bar{r}_j \equiv E[R_j] = \sum_{i=1}^{m} p_i r_{ij} \tag{4}$$

and the mean value $E(\boldsymbol{x})$ and variance $V(\boldsymbol{r})$ derived from the data are as follows:

$$\begin{aligned} E(\boldsymbol{x}) &= \sum_{j=1}^{n} r_j x_j = \sum_{i=1}^{m} p_i \left(\sum_{j=1}^{n} r_{ij} x_j \right) \\ V(\boldsymbol{x}) &= \sum_{i=1}^{m} p_i \left(\sum_{j=1}^{n} r_{ij} x_j - E(\boldsymbol{x}) \right)^2 \\ &= \sum_{i=1}^{m} p_i \left(\left(\sum_{j=1}^{n} r_{ij} x_j \right) - \sum_{i=1}^{m} p_i \left(\sum_{j=1}^{n} r_{ij} x_j \right) \right)^2 = \sum_{k=1}^{n} \sum_{j=1}^{n} \sigma_{jk} x_j x_k \end{aligned} \tag{5}$$

To simplify of the following discussion, we assume each probability p_i to become same value $1/m$. From above-mentioned parameters, we transformed the Markowitz model into the following problem:

$$\begin{aligned} \text{Minimize} \quad & \frac{1}{m} \sum_{i=1}^{m} \left(\sum_{j=1}^{n} (r_{ij} - \bar{r}_j) x_j \right)^2 \\ \text{subject to} \quad & \bar{r}_j = \frac{1}{m} \sum_{i=1}^{m} r_{ij}, \\ & \sum_{j=1}^{n} \bar{r}_j x_j \geq r_G, \quad \sum_{j=1}^{n} a_j x_j \leq b, \\ & x_j \geq 0, \quad j = 1, 2, \ldots, n \end{aligned} \tag{6}$$

Furthermore, introducing parameters $z_i = \sum_{j=1}^{n} (r_{ij} - \bar{r}_j) x_j$, $(i = 1, \ldots, m)$, we equivalently transformed problem (6) into the following problem:

$$\begin{aligned} \text{Minimize} \quad & \frac{1}{m} \sum_{i=1}^{m} z_i^2 \\ \text{subject to} \quad & z_i - \sum_{j=1}^{n} (r_{ij} - \bar{r}_j) x_j = 0, \quad i = 1, 2, \ldots, m \end{aligned}$$

$$\begin{aligned}&\bar{r}_j = \frac{1}{m}\sum_{i=1}^{m} r_{ij},\\ &\sum_{j=1}^{n} \bar{r}_j x_j \geq r_G, \quad \sum_{j=1}^{n} a_j x_j \leq b,\\ &x_j \geq 0, \quad j = 1, 2, \dots, n\end{aligned} \tag{7}$$

Since this problem is a quadratic programming problem not to include the variances, we solve it more efficiently than original Markowitz model setting each parameter.

On the other hand, Konno [8, 9] has proposed Mean-absolute deviation model for portfolio selection problems. Now, let the absolute deviation function:

$$W\left[R\left(x\right)\right] \equiv \sum_{i=1}^{m} p_i \left| \sum_{j=1}^{n} r_{ij} x_i - \bar{r}_p x_i \right| \tag{8}$$

If the returns of assets occur according to a multivariate normal distribution, $\boldsymbol{x}^t \mathbf{V} \boldsymbol{x} = \frac{\pi}{2}\left(W\left[R\left(\boldsymbol{x}\right)\right]\right)^2$ holds based on the result obtained by Konno [8, 9] by using a property of normal distribution with respect to the relationship between this Mean-absolute deviation $W\left[R\left(\boldsymbol{x}\right)\right]$ and the variance. From this formula, problem (7) is transformed into the following problem:

$$\begin{aligned}\text{Minimize} \quad & \frac{\pi}{2}\left(\frac{1}{m}\sum_{i=1}^{m}\left|\sum_{j=1}^{n}\left(r_{ij} - \bar{r}_j\right) x_j\right|\right)^2\\ \text{subject to} \quad & \bar{r}_j = \frac{1}{m}\sum_{i=1}^{m} r_{ij}\\ & \sum_{j=1}^{n} \bar{r}_j x_j \geq r_G, \quad \sum_{j=1}^{n} a_j x_j \leq b,\\ & x_j \geq 0, \quad j = 1, 2, \dots, n\end{aligned} \tag{9}$$

where $p_i = 1/m$, $(i = 1, 2, \dots, m)$. Since the absolute deviation is positive, problem (9) is equivalently transformed into the following problem:

$$\begin{aligned}\text{Minimize} \quad & \frac{1}{m}\sum_{i=1}^{m}\left|\sum_{j=1}^{n}\left(r_{ij} - \bar{r}_j\right) x_j\right|\\ \text{subject to} \quad & \bar{r}_j = \frac{1}{m}\sum_{i=1}^{m} r_{ij}\\ & \sum_{j=1}^{n} \bar{r}_j x_j \geq r_G, \quad \sum_{j=1}^{n} a_j x_j \leq b,\\ & x_j \geq 0, \quad j = 1, 2, \dots, n\end{aligned} \tag{10}$$

Introducing parameters $z_i = \sum_{j=1}^{n}\left(r_{ij} - \bar{r}_j\right) x_j$, problem (10) is transformed into the following problem based on the result of the previous study of Konno [8, 9]:

$$\begin{aligned}\text{Minimize} \quad & \frac{1}{m}\sum_{i=1}^{m}\left|\sum_{j=1}^{n}\left(r_{ij} - \bar{r}_j\right) x_j\right|\\ \text{subject to} \quad & \bar{r}_j = \frac{1}{m}\sum_{i=1}^{m} r_{ij}\end{aligned}$$

$$\begin{aligned}&\sum_{j=1}^{n} \bar{r}_j x_j \geq r_G, \quad \sum_{j=1}^{n} a_j x_j \leq b, \\ &x_j \geq 0, \quad j = 1, 2, \ldots, n\end{aligned} \tag{11}$$

This problem is equivalent to a linear programming problem, and it is more efficiently solved than quadratic programming problem. Consequently, by using this Mean-absolute deviation model, we easily solve a large scale portfolio selection problem. Therefore, in this paper, we focus on the Mean-absolute deviation model.

3. **SRI-based Portfolio Selection Problems with Fuzzy Numbers.**

In previous researches, each return of the scenarios is considered as a fixed value derived from parameters of random variables. However, considering that SRI is usually presented as text-based information, these interpretations are influenced by decision makers' psychological factors, and thus it is difficult to predict the future return as the only fixed value. Therefore, in addition to randomness, we need to consider that the future return is ambiguous. Consequently, we consider several types of portfolio models considering SRI from the aspects of both randomness and fuzziness.

In this study, we focus on a negative SRI screening, which means removing unfavourable investment targets for each investor's optimal value. In order to present the evaluation value of each investment destination with respect to SRI, we introduce a fuzzy number $\tilde{s}_j$ characterized by the following linear membership function:

$$\mu_{\tilde{s}_j}(\omega) = \begin{cases} 1 & 0 \leq \omega \leq S_j^L \\ \dfrac{S_j^U - \omega}{S_j^U - S_j^L} & S_j^L \leq \omega \leq S_j^U \\ 0 & S_j^U \leq \omega \end{cases} \tag{12}$$

This fuzzy number means that S_j^L is the standard negative evaluation value and S_j^U is the worst value conceivable for SRI in terms of the negative screening, i.e., if the values of S_j^L and S_j^U are large, the investor has the negative impression of the investment destination.

Furthermore, coupled with the negative evaluation value, we set the limited value for each purchasing volume x_j as follows:

$$0 \leq x_j \leq X(\tilde{s}_j) \tag{13}$$

where $X(\tilde{s}_j)$ is some monotonous increasing function of $\tilde{s}_j$. Using this fuzzy number with respect to SRI and limited value $X(\tilde{s}_j)$, we propose three types of portfolio models.

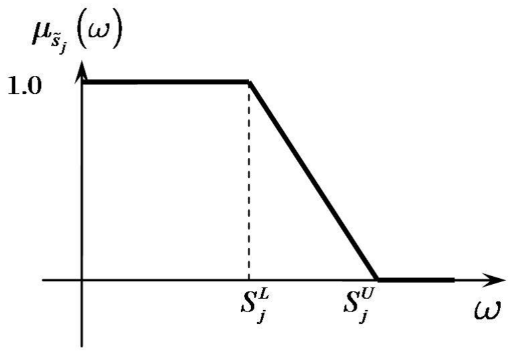

FIGURE 1. Membership function $\mu_{\tilde{s}_j}(\omega)$

3.1. **Direct evaluation approach of SRI.** First, we propose an SRI-based portfolio model evaluating the SRI using fuzzy number (12) directly. Since this model considers minimizing the total negative evaluation value as well as minimizing the total absolute deviation, it is formulated as follows:

$$\begin{aligned}
&\text{Minimize} && \frac{2}{m}\sum_{i=1}^{m} z_i \\
&\text{Minimize} && \sum_{j=1}^{n} \tilde{s}_j x_j \\
&\text{subject to} && z_i + \sum_{j=1}^{n} r_{ij}x_j \geq \sum_{j=1}^{n} \bar{r}_j x_j, \quad i = 1, 2, \ldots, m \\
&&& \bar{r}_j = \frac{1}{m}\sum_{i=1}^{m} r_{ij}, \\
&&& \sum_{j=1}^{n} r_j x_j \geq r_G, \quad \sum_{j=1}^{n} a_j x_j \leq b, \\
&&& 0 \leq x_j \leq X(\tilde{s}_j), \quad j = 1, 2, \ldots, n
\end{aligned} \tag{14}$$

This problem is a bi-criteria and fuzzy programming problem, and so it is hard to solve it directly. Therefore, in order to solve this problem analytically, we introduce a goal programming approach using fuzzy goals. In this paper, fuzzy goals of the total absolute deviation and the total negative evaluation value are formulated as the following linear membership function:

$$\begin{aligned}
\mu_{\tilde{G}_{AD}}(\omega) &= \min\left\{\frac{\sigma_U - \omega}{\sigma_U - \sigma_L}, 1\right\}, \\
\mu_{\tilde{G}_{SRI}}(\omega) &= \min\left\{\frac{g_s^U - \omega}{g_s^U - g_s^L}, 1\right\},
\end{aligned} \tag{15}$$

where parameters σ_L, σ_U, g_s^L and g_s^U are fixed values decided by the decision maker. Furthermore, using a concept of necessity measure, we introduce the degree of necessity with respect to SRI as follows:

$$\begin{aligned}
&N_{\tilde{Z}}\left(\tilde{G}\right) = \inf \max\left\{1 - \mu_{\tilde{Z}}(\omega), \mu_{\tilde{G}_{SRI}}(\omega)\right\}, \\
&\left(\tilde{Z} = \sum_{j=1}^{n} \tilde{s}_j x_j\right)
\end{aligned} \tag{16}$$

Therefore, problem (14) is transformed into the following problem:

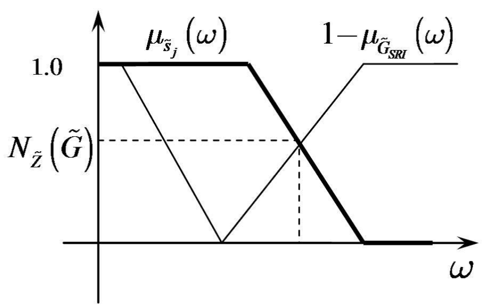

FIGURE 2. Degree of necessity $N_{\tilde{Z}}\left(\tilde{G}\right)$

$$\begin{aligned}
&\text{Maxmize} \quad h \\
&\text{subject to} \quad \mu_{\tilde{G}_{AD}}\left(\frac{2}{m}\sum_{i=1}^{m} z_i\right) \geq h, \quad N_{\tilde{S}}\left(\tilde{G}\right) \geq h, \\
&\qquad z_i + \sum_{j=1}^{n} r_{ij}x_j \geq \sum_{j=1}^{n} \bar{r}_j x_j, \quad \bar{r}_j = \frac{1}{m}\sum_{i=1}^{m} r_{ij}, \quad i = 1, 2, \ldots, m \\
&\qquad \boldsymbol{x} \in \mathbf{X}(h) \triangleq \left\{ \boldsymbol{x} \left| \begin{array}{l} \sum_{j=1}^{n} \bar{r}_j x_j \geq r_G, \ \sum_{j=1}^{n} a_j x_j \leq b, \\ 0 \leq x_j \leq X\left(s_j^h\right), \quad j = 1, 2, \ldots, n \end{array} \right. \right\}
\end{aligned} \tag{17}$$

where we assume that $X\left(s_j^h\right)$ is the limited purchasing volume based on the aspiration level, and it is a constant value to fixed h. Performing the equivalent transformations based on the fuzzy programming, problem (17) is equivalently transformed as follows:

$$\begin{aligned}
&\text{Maxmize} \quad h \\
&\text{subject to} \quad \frac{2}{m}\sum_{i=1}^{m} z_i \leq h\sigma_L + (1-h)\,\sigma_U, \\
&\qquad \sum_{j=1}^{n} S_j^U x_j + h\sum_{j=1}^{n}\left(S_j^L - S_j^U\right) x_j \leq h g_s^U + (1-h)\, g_s^L, \\
&\qquad z_i + \sum_{j=1}^{n} r_{ij}x_j \geq \sum_{j=1}^{n} \bar{r}_j x_j, \quad \bar{r}_j = \frac{1}{m}\sum_{i=1}^{m} r_{ij}, \quad i = 1, 2, \ldots, m \\
&\qquad \boldsymbol{x} \in \mathbf{X}(h)
\end{aligned} \tag{18}$$

3.2. **Case of absolute deviation with SRI-based fuzzy programming approach.** Second, we propose a portfolio model that fuzzy numbers are included in the total absolute deviation in terms of SRI. This model considers minimizing the total absolute deviation with the penalty derived from the negative evaluation value of SRI, and it is formulated as follows:

$$\begin{aligned}
&\text{Minimize} \quad \left(\frac{2}{m}\sum_{i=1}^{m} z_i\right) + f\left(\sum_{j=1}^{n} \tilde{s}_j x_j\right) \\
&\text{subject to} \quad z_i + \sum_{j=1}^{n} r_{ij}x_j \geq \sum_{j=1}^{n} \bar{r}_j x_j, \quad i = 1, 2, \ldots, m \\
&\qquad \bar{r}_j = \frac{1}{m}\sum_{i=1}^{m} r_{ij}, \\
&\qquad \boldsymbol{x} \in \mathbf{X}(h)
\end{aligned} \tag{19}$$

where $f(x)$ is the penalty function with respect to SRI. In a way similar to introducing fuzzy goal (15) and performing the equivalent transformations from (17) to (18), problem (19) is transformed into the following problem:

$$\begin{aligned}
&\text{Maxmize} \quad h \\
&\text{subject to} \quad \left(\frac{2}{m}\sum_{i=1}^{m} z_i\right) + f\left(\sum_{j=1}^{n} \tilde{s}_j x_j\right) \leq h\sigma_L + (1-h)\,\sigma_U, \\
&\qquad z_i + \sum_{j=1}^{n} r_{ij}x_j \geq \sum_{j=1}^{n} \bar{r}_j x_j, \quad i = 1, 2, \ldots, m
\end{aligned}$$

$$\bar{r}_j = \frac{1}{m}\sum_{i=1}^{m} r_{ij}, \qquad (20)$$
$$\boldsymbol{x} \in \mathbf{X}(h)$$

3.3. **Case of fuzzy scenario returns considering SRI.** Third, we consider a SRI-based portfolio model with fuzzy scenario return. By considering each investor's subjectivity for negative evaluation value, we assume that future return is uniformly expected smaller than scenarios derived from the random simulation. This problem is formulated as the following problem:

$$\begin{array}{ll} \text{Minimize} & \dfrac{2}{m}\displaystyle\sum_{i=1}^{m} z_i \\ \text{subject to} & z_i + \displaystyle\sum_{j=1}^{n} \left(r_{ij} - r\left(\tilde{s}_j\right)\right) x_j \geq \displaystyle\sum_{j=1}^{n} \bar{r}_j x_j, \quad i = 1, 2, \ldots, m \\ & \bar{r}_j = \dfrac{1}{m}\displaystyle\sum_{i=1}^{m} r_{ij}, \\ & \boldsymbol{x} \in \mathbf{X}(h) \end{array} \qquad (21)$$

where $r(\tilde{s}_j)$ is the penalty value associated with the negative evaluation value to each investment destination.

In these cases that penalty functions $f(x)$ and $r(\tilde{s}_j)$ are linear, these three problems (18), (20) and (21) are equivalent to linear programming problems on fixed value $h = \bar{h}$. Therefore, we solve these problems analytically using the following solution algorithm based on linear programming approach and bisection algorithm.

Solution algorithm

STEP 1: Elicit the membership function of a fuzzy goal with respect to the total absolute deviation and negative evaluation value. Furthermore, set return scenarios, target values for the total expected value, and monotonous increasing function $X(\tilde{s}_j)$.

STEP 2: Set $h \leftarrow 1$ and solve problem (18). If a feasible solution x exists, then terminate. In this case, the obtained current solution is an optimal solution of main problem.

STEP 3: Set $h \leftarrow 0$ and solve problem (18). If a feasible solution x does not exist, then terminate. In this case, there is no feasible solution and it is necessary to reset a fuzzy goal with respect to the total expected return and variance.

STEP 4: Set $h_L \leftarrow 0$ and $h_U \leftarrow 1$.

STEP 5: Set $h \leftarrow \dfrac{h_L + h_U}{2}$.

STEP 6: Solve problem (18) and find the optimal solution $\boldsymbol{x}(h)$. Then, if $h_U - h_L \leq \varepsilon$ holds with respect to a sufficiently small number ε, $\boldsymbol{x}(h)$ is the optimal solution of main problem, and terminate this algorithm. If not, go to Step 7.

STEP 7: If an optimal solution exists, then set $h_L \leftarrow h$ and return to Step 5. If not, then set $h_U \leftarrow h$ and return to Step 5.

In a way similar to this solution algorithm, we can develop solution algorithms for problems (20) and (21).

4. **Numerical Example.**

4.1. **Comparing the proposed models and mean-absolute deviation model.**

In order to compare our SRI-based proposed models with a standard Mean-absolute deviation model for portfolio selection problems, let us consider an example shown in Table 1 based on securities on the Tokyo Stock Exchange. Let us consider ten securities whose mean values and standard deviations (STD) are based on historical data in five years between 2000 and 2004.

Table 1. Security data (Tokyo stock exchange)

Security	Mean	STD	SRI
S1	0.235	0.265	B
S2	0.388	0.342	A
S3	0.179	0.217	A
S4	0.132	0.095	A
S5	0.210	0.130	C
S6	0.157	0.172	A
S7	0.136	0.145	B
S8	0.291	0.231	B
S9	0.303	0.180	C
S10	0.346	0.198	C

In Table 1, ranks of SRI are three levels A, B and C, which are assigned with the following membership functions:

$$\mu_{\tilde{s}_j}^{A}(\omega)=\begin{cases} 1 & 0\le\omega\le 1 \\ 1-\omega & 1\le\omega\le 2 \\ 0 & 2\le\omega \end{cases},\quad \mu_{\tilde{s}_j}^{B}(\omega)=\begin{cases} 1 & 0\le\omega\le 3 \\ 1-\omega & 3\le\omega\le 4 \\ 0 & 4\le\omega \end{cases},$$

$$\mu_{\tilde{s}_j}^{C}(\omega)=\begin{cases} 1 & 0\le\omega\le 5 \\ 1-\omega & 5\le\omega\le 6 \\ 0 & 6\le\omega \end{cases}$$

Then, let $f(\tilde{s}_j)$ and $r(\tilde{s}_j)$ be

$$f(\tilde{s}_j)=\begin{cases} 0.1\sigma_j\,(\text{Alevel}) \\ 0.2\sigma_j\,(\text{Blevel}) \\ 0.3\sigma_j\,(\text{Clevel}) \end{cases},\quad r(\tilde{s}_j)=\begin{cases} 0\,(\text{Alevel}) \\ 0.1r_{ij}\,(\text{Blevel}) \\ 0.2r_{ij}\,(\text{Clevel}) \end{cases}$$

We set $\sigma_L=0.2$, $\sigma_U=0.3$, $g_s^L=4$, $g_s^U=4.5$, $r_G=0.2$, and the following feasible region:

$$\mathbf{X}(h)\triangleq\left\{\boldsymbol{x}\;\middle|\;\begin{array}{l}\displaystyle\sum_{j=1}^{n}\bar{r}_j x_j\ge r_G,\quad \sum_{j=1}^{n}x_j=1,\\ \displaystyle 0\le x_j\le\frac{1}{(1-h)S_j^L+hS_j^U},\\ j=1,2,\ldots,n\end{array}\right\}$$

Then, we solve standard Mean-absolute deviation model (P1), and three proposed models (18; P2), (20; P3) and (21; P4) in the case of purchasing at beginning of 2005 according to each optimal portfolio in Table 2, and obtain the following portfolio performances after 3 months, 6 months, 1 year and 2 years.

From Table 3, proposed portfolio models based on the SRI, particularly P2 and P4, present higher portfolio performances for the long-term investment than for short-term

Table 2. Each optimal portfolio ratio

Security	P1	P2	P3	P4
S1	0.002	0	0	0
S2	0	0	0	0
S3	0	0.178	0.438	0.149
S4	0.227	0.507	0.562	0.512
S5	0.315	0.001	0	0
S6	0	0	0	0
S7	0	0	0	0
S8	0.001	0	0	0
S9	0.028	0	0	0.034
S10	0.427	0.304	0	0.305

Table 3. Portfolio performances

	3 months	6 months	1 year	2 years
P1	0.0199	0.0888	0.4647	0.5923
P2	0.0096	0.0673	0.4787	0.6332
P3	0.0040	0.0369	0.4436	0.5059
P4	0.0075	0.0686	0.4777	0.6506

than the standard mean-absolute deviation model not considering the SRI. Generally speaking, it is often said that the SRI-based portfolio management has potential as the long-term investment, because the CSR presents each company's vision for the future and investors take actions evaluating the CSR. Therefore, we consider that the result in Table 3 backs this common belief for the SRI.

However, with respect to P3, the portfolio performance is worse than the other models. From Table 2, the number of purchasing securities for P3 is smaller than the other models. This means that it is not to avoid the risk and not to earn the total high return, if investors concentrate funding in a few particular securities using investor's evaluation of CSR reports contrary to decentralization of portfolios.

4.2. **Effectiveness of the proposed model in the current market.** As another practical case in order to compare the proposed SRI-based model with some existing useful portfolio models, we provide the most current data of other ten securities in Table 4 different from data in Table 1. All mean values and STD are calculated from historical data from January to December in 2009 at Tokyo Exchange Stock.

In order to represent the effectiveness of the proposed SRI-based model (18), we introduce three useful portfolio models; mean-variance model (MV), probability maximization model (Pro.max), and probability fractile optimization model (Fractile). We solve these four models under $\sigma_L = 0.05$, $\sigma_U = 0.1$, $g_s^L = 3$, $g_s^U = 4$, $r_G = 0.01$, and consider the case where an investor purchases securities at the end of 2009 according to each calculated portfolio. Then, using 100 data samples from September to December in 2010, we calculate the mean value, maximum value, and minimum value of all total returns. Furthermore, we also calculate the rate of positive total returns from these sample data. Table 5 shows these results.

In Table 5, the only proposed model (18) obtains the positive mean value of total returns mean in this current market data. Furthermore, the rate of the positive total returns of the proposed model is also much higher than the other models. These mean that the

TABLE 4. Other security data (Tokyo stock exchange) from 2009 to 2010

Security	Mean	STD	SRI
S11	0.0174	0.0696	A
S12	0.0173	0.0952	B
S13	0.0011	0.0928	C
S14	0.0396	0.1332	A
S15	0.0156	0.1063	B
S16	0.0875	0.1577	A
S17	0.0091	0.1044	C
S18	0.0386	0.1109	A
S19	0.0005	0.0420	C
S20	0.0274	0.0941	A

TABLE 5. Comparing each factor in four problems derived from 100 samples

	MV	Pro.max	Fractile	Proposed
mean value of total returns	–0.0130	–0.0017	–0.0110	0.0020
maximum value in total returns	0.0401	0.0730	0.0475	0.0594
minimum value in total returns	–0.0785	–0.0969	–0.0717	–0.0896
rate of positive total returns (percent)	41	51	43	62

investor to construct a portfolio using the proposed model may certainly earn the higher investment performance in terms of the total return. Therefore, it will be more important and effective to consider the CSR of each company and to use the SRI-based portfolio model than the other previous portfolio models in current and near future markets.

5. **Conclusion.** In this paper, we have proposed several types of portfolio selection problems based on SRI. In order to apprehend the text-based aspect of SRI, we have introduced fuzzy numbers and formulated fuzzy and multi-criteria programming problems. Furthermore, using fuzzy goals and performing equivalent transformations to the main problem, the proposed models have been equivalent to linear programming-based problems. Using practical numerical examples, we have shown that the proposed models are more adapted for the long-term investment than for short-term, and more effective to the current market.

There are some future works such as the application of the other portfolio models, large-scale and multi-period models, and other fuzzy programming approaches. Furthermore, we will consider the other evaluation of CSR reports such as positive screening and develop more precise and effective evaluation methods.

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