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RELIABILITY ENHANCEMENT OF A TRAFFIC SIGNAL LIGHT SYSTEM USING A MEAN-VARIANCE APPROACH

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Abstract. *Traffic accidents cause tragic loss of life, property damage and substantial congestion to transportation systems. A large percentage of crashes occur at or near intersections. Therefore, traffic signals are often used to improve traffic safety and operations. The objective of this study is to present a significant and effective method of determining the optimal investment involved in retrofitting signals with light emitting diode (LED) units. In this study, the reliability and risks of each unit are evaluated using a variance-covariance matrix, and the effects and expenses of replacement are analyzed. The mean-variance analysis is formulated as a mathematical program with the objectives of minimizing the risk and maximizing the expected return. Finally, a structural learning model of a mutual connection neural network is proposed to solve problems defined by mixed-integer quadratic programming, and this model is employed in the mean-variance analysis. Our method is applied to an LED signal retrofitting problem. This method enables us to select results more effectively and enhance decision-making.*
Keywords: Boltzmann machine, Hopfield network, Mean-variance analysis, Structural learning

1. Introduction.

Traffic accidents cause tragic loss of life, property damage and substantial congestion to national transportation systems. A large percentage of traffic accidents occur at or near intersections [1]. Pernia indicates that intersection-related crashes are responsible for very high percentage of the total number of crashes in the roadway system. Traffic signals are often used to improve traffic safety and operations at intersections [2]. Light emitting diodes (LEDs) are often preferred due to their operational and low energy consumption advantages [3]. Despite their excellent performance, several barriers hinder more rapid retrofitting of signals, such as high retrofitting costs and capital constraints. Thus, one of the most significant challenges in deciding a retrofitting strategy is to select which intersection signal to replace with LEDs and which to leave as they are.

Strong progress has been made in artificial neural network research in recent years. Neural networks have been applied to various fields, such as pattern recognition, forecasting, robotic, data mining, multiple-objective decision-making, and combinatorial optimization [4-7]. In this study, we applied a neural network to solve the portfolio selection problem efficiently. Boltzmann machines (BMs) are interconnected neural networks first proposed by Hinton [8]. These machines represent an improvement on the Hopfield network, which uses a probability rule to update the state of a neuron and its energy function. Thus, the energy function of a BM hardly ever falls into a local minimum.

In this study, we propose an investment decision method that uses data regarding the accident number at intersections. Mean-variance analysis addresses the mathematical problem of allocating a given amount of money among several alternative investments. Here, we define a portfolio as an investment made at certain intersections using a given amount of money. Markowitz originally proposed and formulated the mean-variance approach based on the portfolio selection problem [9,10]. This method is formulated as a mixed-integer quadratic programming problem. We formulate a two-layered neural network comprising both a Hopfield network and a BM to effectively and efficiently select a limited number of units from those available. The Hopfield network is employed in the upper layer to select a limited number of units, and the BM is used in the lower layer to determine the optimal solution/units from the limited number of units selected by the upper layer. In this study, by building a double-layered BM, both layers are optimally configured by structural learning. The results obtained enable us to reduce the computational time and cost, and to more easily understand the internal structure.

The following section briefly introduces mean-variance analysis. Section 3 introduces the Hopfield and the BM, and Section 4 explains the double-layered BM. Section 5 provides descriptions of LED signal retrofitting and the mean-variance problem, and presents a numerical example to illustrate these. In Section 6, the practical use and the simulation results are discussed. Finally, conclusions are presented in Section 7.

2. Mean-Variance Analysis.

Mean-variance analysis, originally proposed by H. Markowitz during the early 1950s [9], is a widely used investment theory. It assumes that most decision-makers have an aversion to risk, even if this lowers their expected return. However, it is difficult to identify a utility function because individual decision-makers have different utility structures. Hence, Markowitz formulated mean-variance analysis as the following quadratic programming problem, under the restriction that the expected return rate must be greater than a specified amount.

In this study, the following formulation has been developed.

$$\text{maximize} \quad \sum_{i=1}^{n} \mu_i m_i x_i \tag{1}$$

$$\text{minimize} \quad \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} m_i x_i m_j x_j \tag{2}$$

$$\text{subject to} \quad \sum m_i x_i = 1 \tag{3}$$

$$\sum_{i=1}^{n} m_i = S \tag{4}$$

$$m_i \in \{0,1\} \ (i = 1, 2, \ldots, n) \tag{5}$$

$$x_i \geq 0 \ (i = 1, 2, \ldots, n) \tag{6}$$

Here, S denotes the desired number of units to be selected in the portfolio, m_i is the decision variable for unit i (m_i is 1 if any unit i is held, 0 otherwise), σ_{ij} is the covariance between units i and j, μ_i is the expected return rate of unit i, and x_i is the investment rate for unit i. The developed formulation is a mixed integer quadratic programming problem with two objective functions, i.e., the expected return rate and the degree-of-risk. In mixed integer quadratic programming, it is almost impossible to obtain the optimal solution from a large set of possible combinations. Thus, this study intends to obtain the optimal solution to this problem by employing the combination of a Hopfield network and a BM.

3. **Hopfield Network and Boltzmann Machine.** The Hopfield network is a fully connected, recurrent neural network, which uses a form of the "generalized Hebb rule" to store Boolean vectors in its memory. In any situation, combining the state of all units leads to a global state for the network. This global state is the input, together with other prototypes, which are stored in the weight matrix based on Hebb's postulate, formulated as

$$w_{ij} = \frac{1}{P}\sum_{p=1}^{P} x_i^p x_j^p \tag{7}$$

where w_{ij} is the weight of the connection from neuron j to neuron i, P is the number of training patterns and x_i^p is the p-th input for neuron i. The Hopfield network cab be use to minimize an energy function during its operation. The simplest form of the energy function is given by the following:

$$E = \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{N} w_{ji} x_j x_i \tag{8}$$

Here, w_{ij} denotes the strength of the influence of neuron j on neuron i. The weights w_{ij} are created using Hebb's postulate as mentioned above and belong to a symmetric matrix with the main diagonal containing only zeroes. Because of this useful property, the Hopfield network can also be used to solve combinatorial optimization problems. However, Hopfield networks suffer from the major disadvantage that they sometimes converge to a local rather than the global minimum. To overcome this problem, a modification was made to the BM [8].

The BM can be seen as a stochastic, generative counterpart of the Hopfield network. In the BM, probability rules are employed to update the state of neurons and the energy function as follows:

If $V_i(t+1)$ is the output of neuron i in the subsequent time iteration $t+1$, then $V_i(t+1)$ is 1 with probability P and 0 with probability $1-P$, where

$$P = f\left(\frac{u_i(t)}{T}\right) \tag{9}$$

Here, $f(\cdot)$ is a sigmoid function, $u_i(t)$ is the total input to neuron i shown in (10), T is the network temperature, and

$$u_i(t) = \sum_{j=1}^{N} w_{ij} V_i(t) + \theta_i \tag{10}$$

where w_{ij} is the weight of the connection from neurons i to neuron j, θ_i is the threshold of neuron i and V_i is the state of unit i. The energy function, E, is written as follows:

$$E = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij} V_i V_j - \sum_{i=1}^{N} \theta_i V_i \tag{11}$$

In this study, the combination of a Hopfield network and a BM offers a solution to the problem of finding the optimal number of units in the neural network. Accordingly, this study proposes a double-layered BM, which we discuss in detail in the following section.

4. **Double-Layered Boltzmann Machine.** Following is a brief an explanation on how to transform the mean-variance model formulated as the energy function of the BM [11,12].

The objective function is transformed, shown as (1) and (2), into the energy function in (11)

$$E = -\frac{1}{2}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_ix_j + 2\sum_{i=1}^{n}\sum_{j=1}^{n}x_ix_j\right) + 2\sum_{i=1}^{n}x_i + K\sum_{i=1}^{n}\mu_ix_i \tag{12}$$

where K is a real number not less than 0. If K is set to a large value, the expected return is emphasized much more than the risk. If $K = 0$, the BM converges to the problem of minimizing the risk. When the energy function of the BM described in this section converges to the global minimum, the investment rate for each intersection is obtained as the output of each unit.

The double-layered BM model deletes the units of the lower layer, which are not selected in the upper layer, in its execution. Then the lower layer is restructured using the selected units. Because of this feature, the double-layered BM converges more efficiently than a conventional BM. The double-layered BM just described converts the objective function into the energy functions of two components, namely the upper layer, E_u, and the lower layer, E_l, as described below.
Upper layer

$$E_u = -\frac{1}{2}\sum_{i=j}^{n}\sum_{j=1}^{n}\sigma_{ij}s_is_j + K_u\sum_{i=1}^{n}\mu_is_i \tag{13}$$

Lower layer

$$E_l = -\frac{1}{2}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_ix_j + 2\sum_{i=1}^{n}\sum_{j=1}^{n}x_ix_j\right) + 2\sum_{i=1}^{n}x_i + K_l\sum_{i=1}^{n}\mu_ix_i \tag{14}$$

Here K_u and K_l are the weights of the expected return rates of the upper and lower layers, respectively, and s_i is the output of the i-th unit of the upper layer.

This paper also proposes a model that allows for noise injection. Therefore, the double-layered BM is tuned such that the upper layer influences the lower layer with weight 0.9, and the lower layer influences the upper layer with weight 0.1. The double-layered BM is iterated with

$$U_i = 0.9y_i + 0.1x_i \tag{15}$$

for the upper layer, and

$$L_i = x_i(0.9y_i + 0.1) \tag{16}$$

for the lower layer, where U_i and L_i denote noise-injected stimulus. Here, the value U_i in the upper layer is transferred to the corresponding nodes in the upper layer, and the value L_i in the lower layer is a value transferred to the corresponding nodes in the lower layer. The value y_i represents the present state of node i in the upper layer, and x_i is the value of the present state of node i in the lower layer. A value for L_i would indicate that 90% of the value is determined from the value of node i in the upper layer. When U_i is 1, $L_i = x_i$; otherwise, when y_i is 0, 10% of the value of x_i is transferred to the other nodes. On the other hand, U_i has an influence of 10% on the lower layer. Therefore, even if the upper layer converges to a local minimum, the disturbance from the lower layer causes the upper layer to escape from this local minimum. When the local minimum possesses a large energy barrier, dynamic behavior may be used (by changing 0.9 and 0.1 dynamically); this phenomenon is similar to simulated annealing where many proofs of convergence of simulated annealing in the literature [13].

Figure 1 illustrates the structure and the algorithm of the double-layered BM is as follows:

Step 1: Set the number of units
Set the initial value of each unit
Set the value h
Set the start, restructure and end of the control parameter T (temperature)
Set the control parameter update frequency;
Step 2: Input K_u and K_l;
Step 3: Execute the upper layer (start running the Hopfield network in the upper layer);
Step 4: If the output value of a unit in the upper layer is 1, then add h to the corresponding unit in the lower layer. Execute the lower layer;
Step 5: After executing the lower layer at a constant frequency, decrease the temperature;
Step 6: If the output value for certain units are sufficiently large, then add h to the corresponding unit in the upper layer;
Step 7: Iterate from Step 3 to Step 6 until the temperature reaches the restructuring temperature;
Step 8: Restructure the lower layer using the selected units in the upper layer;
Step 9: Execute the lower layer until the termination condition is reached.

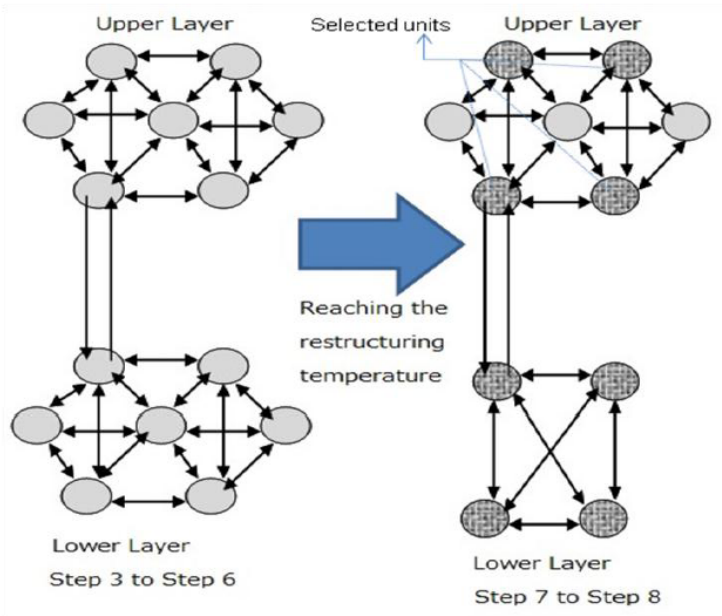

FIGURE 1. Double-layered BM

5. **Numerical Examples of LED Signal Retrofitting.** Ten intersections areas, for which accident data from the previous ten years were available, were chosen to analyze and optimize portfolios of retrofitting. In this study, the trade-off between the mean number of accidents and variance was analyzed using mean-variance analysis and a double-layered BM, which becomes considerably more efficient as the number of intersections increases. The simulation parameters are employed in the following steps:
Upper layer:

1: The change is carried out using 0.001 inter-arrival temperatures.
2: Each unit is set to an initial value of 0.1.

3: The constant K is simulated taking the values 0.3, 0.5, 0.7 and 1.0.

Lower layer:

1: The temperature T of the BM is changed from 100 to 0.0001.
2: The change is carried out using 0.001 inter-arrival temperatures.
3: Each unit is set to an initial value of 0.1.
4: The constant K is simulated taking the value 0.3, 0.5, 0.7 and 1.0.
5: As the BM behaves probabilistically, the result is taken to be the average of the final 10, 000 times.

The proposed method is implemented using the following five steps:

Step 1. Identifying the Correct Uncertainty – Security

Identifying the correct uncertainties is the first step in mean-variance analysis. Uncertainty characteristics that that differentiates one asset from another should be included in the analysis. Security is just one source of differentiating uncertainty; policy, market conditions and manufacturing capability are others.

Step 2. Quantifying Individual Uncertainties – Accident numbers

Uncertainty identified based on security is quantified using accident data from the previous ten years. Table 1 shows the historical accident numbers of ten intersections. Other uncertainties might not be so straightforward to quantify.

Table 1. Accident numbers at intersections

	‘00	‘01	‘02	‘03	‘04	‘05	‘06	‘07	‘08	‘09	Mean
IS1	11	15	2	8	10	7	6	13	11	9	9.20
IS2	5	3	7	5	6	8	10	4	2	2	5.20
IS3	11	13	2	8	7	5	8	7	10	11	8.20
IS4	3	6	7	1	4	5	6	3	5	2	4.20
IS5	1	6	7	8	10	4	7	5	10	6	6.40
IS6	3	4	9	4	4	3	4	5	7	5	4.80
IS7	15	13	10	6	4	9	7	10	14	8	9.60
IS8	3	8	4	9	6	7	8	6	3	6	6.00
IS9	2	10	15	14	3	4	11	9	12	8	8.80
IS10	6	5	7	8	8	6	8	7	4	9	6.80

Step 3. Post-processing the Uncertainties – Covariance matrix

The covariance matrix represents the relative independence and uncertainty of the assets. The matrix is created by placing the variance of the assets on the diagonal and the pair-wise covariance (calculated using (17)) on the off-diagonals. Table 2 shows the covariance matrix of the 10 intersections that were used in the case study described here.

$$\sigma_{x_1,x_2} = \rho_{x_1,x_2}\sigma_{x_1}\sigma_{x_2} \tag{17}$$

Step 4. Applied Mean-variance Theory – Mean-variance analysis

To enable the decision maker to retrofit LED signals a set of assets is used that maximize return while considering risk aversion. The specific class of optimization used is quadratic optimization, which is based on an appropriate balance of risk and return. Such risks and returns are typically derived from historical accident numbers at the intersections. The quadratic programming problem can be solved using the mean-variance analysis method, which employs double-layered BM efficiently.

As shown in Table 3, given $K = 0.3$, IS2 should be allocated 21.4 percent, IS4 21.1 percent, IS5 20.1 percent, IS7 13.7 percent and IS8 23.7 percent, out of the total budget. Other intersections, which are not included in the list of units after restructuring, should

Table 2. Covariance matrix representing 10 intersections

	IS1	IS2	IS3	IS4	IS5	IS6	IS7	IS8	IS9	IS10
IS1	18.16	–5.04	15.36	–1.64	–3.08	–7.56	5.28	3.40	–12.16	–2.76
IS2	–5.04	1.76	–4.64	0.36	1.12	1.84	–2.32	–1.60	0.84	1.04
IS3	15.36	–4.64	14.16	–2.24	–5.08	–6.76	7.28	2.60	–9.16	–2.76
IS4	–1.64	0.36	–2.24	4.56	0.32	3.04	2.68	–1.60	2.44	–1.16
IS5	–3.08	1.12	–5.08	0.32	9.04	1.68	–11.24	3.80	5.48	2.28
IS6	–7.56	1.84	–6.76	3.04	1.68	4.56	–0.68	–1.40	7.56	0.36
IS7	5.28	–2.32	7.28	2.68	–11.24	–0.68	17.04	–4.20	–3.28	–4.28
IS8	3.40	–1.60	2.60	–1.60	3.80	–1.40	–4.20	5.20	5.20	0.40
IS9	–12.16	0.84	–9.16	2.44	5.48	7.56	–3.28	5.20	29.36	0.76
IS10	–2.76	1.04	–2.76	–1.16	2.28	0.36	–4.28	0.40	0.76	1.36

Table 3. Simulation result in investment rate for each intersection

Intersection	$K = 0.3$	$K = 0.5$	$K = 0.7$	$K = 1.0$
IS1				
IS2	0.214	0.253	0.271	0.297
IS3				
IS4	0.211	0.097	0.052	0.038
IS5	0.201	0.207	0.155	0.041
IS6				0.027
IS7	0.137	0.179	0.198	0.213
IS8	0.237	0.251	0.264	0.278
IS9		0.013	0.046	0.079
IS10			0.014	0.027

Table 4. Expected return rate and risk

K	Expected Return Rate	Risk
0.3	19.2580	0.0108
0.5	19.3010	0.0129
0.7	19.3970	0.0140
1.0	19.4290	0.0151

not receive any investment. When $K = 0.5$, six intersections were selected in the list of units after restructuring; when $K = 0.7$, seven intersections were selected, and when $K = 1.0$, eight intersections were selected. From this result, it can be concluded that the number of intersections selected is directly proportional to K.

Step 5. Determining the Optimal Maintenance Strategy – Selection of K

Indifference curves typically take the mathematical form shown algebraically in (18). In (18), V represents the expected return or value of a system, where σ^2 is the uncertainty of the portfolio and V_0 is an initial value for the expected return or value of the system when uncertainty is zero.

$$V = V_0 + k\sigma^2 \tag{18}$$

The expected return rate and risk are calculated as shown in Table 4, which also indicates four different levels of risk aversion, the value of K, and reflect various decision

Table 5. Comparison of double-layered and conventional BMs

	Computational Times (sec.)		
No. of Intersections	Conventional BM	Double-layered BM	Computing Efficiency (%)
10	7.75	6.86	11.48
20	11.87	7.59	36.06
40	13.88	8.22	40.78
80	22.01	9.52	56.75
160	41.55	11.61	72.06
320	101.54	16.58	83.67
640	223.33	33.80	84.87

Table 6. Comparison of the double-layered BM and the conventional BM in terms of expected return rates, risk and the number of selected units

	Expected return rate		Risk		No. of Selected Intersections	
No. of Intersections	CBM	DBM	CBM	DBM	CBM	DBM
10	19.2579	19.2580	0.0108	0.0108	5	5
20	19.2599	19.2599	0.0131	0.0131	5	6
40	19.2678	19.2679	0.0199	0.0200	7	7
80	19.2796	19.2797	0.0267	0.0266	6	9
160	19.2887	19.2888	0.0351	0.0349	4	10
320	19.2999	19.3000	0.0419	0.0417	5	12
640	19.3615	19.3617	0.0525	0.0526	6	14

Note: CBM: Conventional BM, DBM: Double-layered BM

maker preferences. When K is set to a larger value, the solution is obtained for high return rates and high risk.

Table 5 compares the double-layered and conventional BMs, employing various sizes from 10 intersections to 640 intersections. Computational efficiency is given by the following equation:

$$C_e = (t_{BM} - t_{DBM})/t_{BM} \times 100 \tag{19}$$

where C_e denotes computational efficiency, t_{DBM} the computation time of the double-layered BM, and t_{BM} the computation time of a conventional BM. The computing time of the double-layered BM is dramatically shorter. This is because the double-layered BM deletes useless units during the restructuring step. In contrast, a conventional BM computes all units until the termination condition is reached. The double-layered BM is therefore more computationally efficient.

Table 6 shows that the conventional BM and double-layered BMs provide similar results in term of expected return rates and risk. For number of selected unit/intersections, the table illustrates that the proposed method is more efficient than the conventional method because when the number of the total unit increases, the number of the selected units also increases. This follows the portfolio theory from the risk aversion a perspective.

6. **Discussions.** LED traffic signals have many advantages over conventional traffic lights including significantly reduced electric power consumption, dramatically lower maintenance costs and improved safety due to greater brightness. Despite their excellent performance, several barriers hinder more rapid retrofitting, such as high retrofit costs and

capital constraints. Thus, one of the most significant challenges in retrofitting strategy is the decision-selecting which intersection to signalized with LEDs, which to keep on the table. We provide a method for selecting intersections that is not based on traditional effectiveness measures such as cost and performance, but rather on a quantitative analysis of the uncertainty embedded in each potential intersection. In this method, cost and performance remain central issues in decision making, but uncertainty serves as the focal point to identify potentially powerful combinations of intersections to explore concurrently in decision phases. We present here a method to identify and quantify uncertainty in intersections and a means to manage it using mean-variance analysis and optimization. Perhaps best known to economists and investors, mean-variance analysis is based around the objective of maximizing return, subject to the risk aversion of the decision maker. This simple concept and the theoretical accuracy with which the theory has been converted to practice are presented as the means of exploring the retrofitting strategy of potential intersections around the main theme of uncertainty.

Due to the requirement for safety at traffic signals, we first and foremost considered accident numbers at intersections. Conventional methods might decide the order of retrofitting intersections based on their accident ranking. In practice, however, more accidents may unexpectedly occur than before in intersections that have not been renewed. Thus, regardless of the order of LEDs selected for retrofitting, uncertainty (risk) will remain. Clearly, the existence of uncertainty has to be considered in the analysis of rational retrofitting strategies.

We studied 10 intersections and their 10-years accident data, from which portfolios of retrofitting could be analyzed and optimized. We proposed an effective retrofit strategy where risk (measured by the variance in accident numbers) is considered together with the mean number of accidents. Our analysis of the trade-off between the mean number of accidents and variance employs mean-variance analysis and double-layered BM, which becomes considerably more efficient as the number of intersections dramatically increases.

It appears obvious that investors are concerned with risk and return and that these parameters should be measured for the portfolio as a whole. Variance (or standard deviation), was considered as a measure of portfolio risk. Because the variance of the portfolio (the variance of a weighted sum) involved all covariance terms, the approach appeared more plausible. Because two criteria existed (i.e., expected return and risk), the natural approach for an economics program was to imagine the investor selecting a point from the set of optimal expected return and variance of return combinations, now known as the efficient frontier. In this section, we employ double-layered BM as an efficient model to solve this trade-off problem.

The implementation of the proposed method uses the following five steps:

Step 1. Identifying the correct uncertainty

Step 2. Quantifying individual uncertainties

Step 3. Post-processing the uncertainty

Step 4. Implementing Portfolio Theory

Step 5. Determining the optimal maintenance strategy

We have demonstrated that proposed method has several advantages. The results show that the double-layered BM can select and determine an optimum investment rate for intersections using less computation time than conventional BMs, thereby decreasing costs. The results also demonstrated that our proposal for incorporating structural learning into the BM is effective and can enhance the reliability of a traffic signal light retrofitting problem.

7. **Conclusions.** In this study, an innovative proposal based on mean-variance analysis was presented to compute the optimal cost investment strategy for a traffic signal system. A double-layered BM was proposed to solve the mean-variance analysis problem. Furthermore, to evaluate the proposed method, a case study was performed to illustrate how the method generates a quantitative prediction of the effective allocation of investment costs in implementing intersection LED signal retrofitting, based on the time-series data available for a ten-year period. The calculation of the costs associated with maintenance becomes as important as the computation of the reliability itself. This study observed that the proposed method can deal with these types of problems much more effectively than conventional methods and demonstrated its effectiveness in dealing with the uncertainties in the case study. The results obtained demonstrated that the selection and investment expense rates of the intersections could be extended to enhance traffic safety. The results also demonstrated that our proposed method is effective and can enhance the decision-making process. The simulation showed that computational times are significantly shorter than those required for a conventional BM and its can be prolonged to increase cost savings.

In the future, we will further investigate the reliability of the proposed method, along the following lines:

1) Because the system is restructured after reaching a certain temperature, the dynamics of the inner structure is important. By studying the inner structure of the double-layered BM, it might be possible to find a way to improve the accuracy of the proposed method.
2) The method will be compared with other method or packages that have been developed, such as ILOG CPLEX, to compare the performance of the proposed method.
3) The proposed method will be applied to various engineering problems.

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