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ON A SMOOTHER FOR DISCRETE-TIME LINEAR STOCHASTIC SYSTEMS WITH UNKNOWN DISTURBANCES

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Abstract. *We consider discrete-time linear stochastic systems with unknown disturbances and study the fixed-point smoothing problem for these systems. A new smoothing algorithm which is robust with respect to the unknown disturbances is derived from the optimal filter with disturbance decoupling property for those systems. It is shown that this algorithm reduces to the standard optimal smoother derived from the Kalman filter when the unknown inputs disappear. This indicates that the new smoothing algorithm is a natural extension of the standard optimal smoother to the linear systems possibly with unknown inputs.*

Keywords: Stochastic systems, Smoother, Optimal filter, Unknown inputs

1. **Introduction.** We consider optimal smoothing problems for discrete-time linear stochastic systems. It is well known that the standard Kalman filter is the optimal linear filter in the sense that it minimizes the mean-square error in an appropriate class of linear filters [21,23,25-27]. We need to recall that the Kalman filter can work well only when we have accurate mathematical modelling of the monitored systems. However, in practice, models derived by engineers very often contain modelling errors which greatly increase state estimation errors as if the models have unknown disturbances.

In order to develop reliable filtering algorithms which are robust with respect to unknown disturbances and modelling errors, many research works have been published based on the disturbance decoupling principle. Pioneering works were done by Darouach et al. [13,14], Chang and Hsu [9] and Hou and Müller [18]. They utilized some transformations to make the original systems with unknown inputs into some singular systems without unknown inputs. The most important previous study related to this paper was done by Chen and Patton [10]. They proposed the simple and useful optimal filtering algorithm, ODDO (Optimal Disturbance Decoupling Observer), and showed its excellent simulation results. See also the papers such as [8], [19], [20] and [30] and the book [11]. Their algorithm recently has been modified by the author in [31] (see [32] also).

We here review some definitions on filtering and smoothing. The optimal filtering problem is to investigate the optimal estimates of the state x_t or x_{t+1} with minimum variance based on the observation $\mathbf{Y}_t$ of the states $\{y_0, y_1, \cdots, y_t\}$, i.e., $\mathbf{Y}_t = \sigma\{y_s, s = 0, 1, \cdots, t\}$ (the smallest σ -field generated by $\{y_0, y_1, \cdots, y_t\}$ (see e.g., Katayama [27], Chapter 4)), but we here consider smoothing problems which allow us time-lags for computing state