International Journal of Innovative Computing, Information and Control Volume 2, Number 5, October 2006

OPTIMAL TRANSMISSION OF A SET OF DISCRETE-TIME GAUSSIAN SIGNALS THROUGH CHANNELS WITH FEEDBACK

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Received December 2005; revised June 2006

ABSTRACT. We consider the problem of optimal transmission of a set of correlated Gaussian signals through parallel channels with feedback. We send the signals by a linear encoding with output feedback. It is well-known that the optimal output feedback which minimizes the power of the encoded signal is given by the least-squares estimate of the linear term. Under a constraint on the total transmission power, we consider the problem of computing a set of gains for the channels which maximizes the mutual information between the output and the signal.

Keywords: Gaussian processes, Optimal transmission, Least-squares state estimation, Kalman filter

1. Introduction. For Gaussian channels with feedback, there are a number of studies on optimal transmission problems [1-9]. Ihara [1] considered a single channel problem where the signal is a scalar random variable and the coding is linear in the signal. In his formulation, the coding is composed of the following two terms:

- (i) a linear function of the signal with a gain coefficient which is a deterministic function of time;
- (ii) a feedback signal which is a functional of the past channel output.

He showed that the optimal value of (ii) which maximizes the mutual information under a constraint on the transmission power is the least-squares estimate of (i), and that the encoded signal is optimal when it is the estimation error multiplied by the gain coefficient. This result is generalized by Liptser and Shiryayev [2] to the case where the signal is a scalar Gaussian stochastic process which is given by a solution of a linear stochastic differential equation. They also showed that even when we allow the gain to be dependent on the output, the optimal one is still independent of the output and is a deterministic time-function. Both in [1] and [2], the optimal gain in (i) is easily computed if we assign the maximum value of the transmission power.

For a multi-dimensional case with parallel transmission channels, Kunita [3] considered the corresponding coding problem with the following components:

- (i') a vector-valued linear function of the signals described by a multiplication of a deterministic gain matrix to the signal vector;
- (ii') a vector-valued feedback signal which is a causal functional of the outputs of the channels.