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SIMULATION ANALYSES OF SPATIO-TEMPORAL PATTERNS FORMED BY CHEMOTACTIC BACTERIA UNDER RANDOM FLUCTUATIONS

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Abstract. *It is well known that a self-organization plays an important role in various fields of engineering. The self-organization can create many kinds of spatio-temporal patterns such as spiral and target patterns in the BZ(Belousov-Zhabotinsky) reaction and a phase separation pattern in a high-polymer substance. In this paper, we focus attention on bacterial colony patterns among many spatio-temporal patterns created by the self-organization and study a mathematical modeling of the chemotactic bacterial colony formation such as Salmonella typhimurium (S. typhimurium) and Escherichia coli (E. coli). Since we cannot analyze the bacterial colony formation under the existence of fluctuations in the concentration of chemical substances and the population density of bacteria by the conventional deterministic model, we propose a stochastic model of chemotactic bacterial colony formations and study the influence of fluctuations on the bacterial colony formations by numerical simulations.*
Keywords: Reaction diffusion equation, Self-organization, Random noise, Chemotaxis, Bacterial colony formations, Numerical simulations

1. **Introduction.** We often observe spatio-temporal patterns created by the self-organization [1,2] in many fields of engineering and in the natural world [1-4]. An analysis of such pattern formation processes is important as a basic problem in relevant fields. For example, an analysis of a phase separation pattern in a high-polymer substance is essential to develop new materials [4]. Among many spatio-temporal patterns generated by the self-organization, we focus attention on the bacterial colony patterns [1-7].

Bacteria such as *Salmonella typhimurium* (*S. typhimurium*) and *Escherichia coli* (*E. coli*) move in the direction of increasing concentration of chemoattractant, asparate. As a result, they arrange themselves into high-density aggregates, i.e., they form bacterial colonies. This property of bacteria is called the chemotaxis. Some mathematical models of bacterial colony formation under the chemotaxis have been proposed in the past in the deterministic framework [1,5,8]. But in the real situation, there exist fluctuations in concentrations of chemical substance caused by impurities and the fluctuation in behavior of bacteria caused by difference of growth rates of each bacterium. In this paper, we propose the stochastic reaction diffusion equations as the model of the chemotactic bacterial colony formation under such fluctuations.

As the way of culturing the bacteria *S. typhimurium* and *E. coli* in experiments, there are cultures in the liquid and the semi-solid media (0.24% water agar). In the liquid media, *S. typhimurium* and *E. coli* form comparatively simple and similar temporary bacterial colony patterns [8,10], whereas in the semi-solid media, they form more complex and different spatio-temporal colony patterns [9], which are radial spots, concentric ring and