International Journal of Innovative Computing, Information and Control
Volume **5**, Number **1**, January **2009**

ICIC International ©2009 ISSN 1349-4198
pp. 75–85

A METHOD OF OPTIMIZATION OF LINEAR OBSERVATIONS FOR THE KALMAN FILTER BASED ON A GENERALIZED WATER FILLING THEOREM

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Received February 2008; revised July 2008

Abstract. *We are concerned with an optimization problem of the gain matrix, over a given time interval, of the linear observation for the Kalman filter. The innovations process included in the Kalman filter has the same structure as the model of a set of parallel transmission channels with the optimal output feedback. In the linear coding problem for this set of channels, it is well-known that the optimal output feedback which minimizes the power of the encoded signal is given by the least-squares estimate of the linear term in the coding and that the channel output then becomes the innovations process. By applying this solution of the optimal transmission problem, we obtain, at any time points, a set of the gains, with non-unique elements, which maximize the mutual information between the observation and the signal under a constraint on the power of the innovations process. Finally, the gain matrix is uniquely determined by an optimization of minimizing estimation error which is local in time.*

Keywords: Gaussian processes, Kalman filter, Least-squares state estimation, Optimal transmission

1. **Introduction.** Starting from 1970's, the optimization problem of observations associated with the Kalman filter has been studied in many literatures. Most of them formulated the problem as a kind of optimal control problem with a quadratic performance criterion. However, because of the existence of the nonlinearity in the Riccati equation of the estimation error covariance matrix, there are few result which is applicable in the real design of the Kalman filter. In this paper, we apply the fact that the innovations process is described by the same equation which was used in the discussion of the optimal transmission problem of Gaussian signals through parallel channels with feedback [1-5]. According to the Shannon's information theory, the observation is considered to be better when the mutual information between the signal and the observation takes a larger value. On the other hand, form the view point of the performance of the Kalman filter, the observation is better when we have a smaller estimation error. Taking into account of these points, our approach in this paper is quite new and by the following two steps:

(i) Information theoretic optimization to maximize the mutual information between the signal and the observation subject to a power constraint concerned with the innovations process;
(ii) Optimization of the performance of the Kalman filter, i.e., minimization of the estimation error variance.

By using this type of approach, we already discussed the optimization problem for the stationary Kalman filter [6] where all the coefficients of the signal and observations equations are time-invariant. In this paper, we are concerned with the optimization of time-varying