International Journal of Innovative Computing, Information and Control
Volume **5**, Number **1**, January **2009**

ICIC International ©2009 ISSN 1349-4198
pp. **87–96**

ON A CLASS OF ONE-DIMENSIONAL CHAOTIC DISCRETE DYNAMICAL SYSTEMS WITH ONE OR TWO STEP TYPE PIECEWISE UNIFORM INVARIANT DENSITY

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Received March 2008; revised July 2008

Abstract. *Nonlinear systems described by the simple mathematical model often exhibit extremely complicated behavior called chaos. In this paper, chaotic systems, described by the one-dimensional difference equation, are investigated. Methods for constructing chaotic systems with the invariant density, which is piece-wise uniform with one or two step, are demonstrated.*

Keywords: Chaos, Invariant measure, One-dimensional nonlinear map, Lyapunov exponent, Frobenius-Perron equation

1. **Introduction.** In various research fields, it is well known that nonlinear systems with a simple mathematical model often exhibit extremely complicated behavior called chaos [1-12]. In this paper, chaotic behavior, exhibited by the following one-dimensional difference equation, is investigated

$$\begin{aligned} x_{n+1} &= f\left(x_n\right), \quad n = 0, 1, 2, \ldots \\ f &: \ I \to I, \qquad I = [0, 1] \end{aligned} \tag{1}$$

Concerned with the nonlinear function $f(x)$, which determines the characteristic of the nonlinear system (1), if a function $p(x)$, $x \in [0, 1]$ satisfies the following Frobenius-Perron equation

$$p(y) = \sum_{x \in f^{-1}(y)} \frac{p(x)}{|f'(x)|}, \quad f'(x) = \frac{df(x)}{dx} \tag{2}$$

and satisfies the following normalized condition

$$\int_I p(x)\, dx = 1$$

then the function $p(x)$ is called the invariant density of the nonlinear function $f(x)$ [5]. Important statistical information of chaotic behavior of the system (1) are included in the invariant density $p(x)$. For example, for any subset $A \subset R^1$, the equation

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} \chi_A(x_k) = \int_I \chi_A(x)\, p(x)\, dx \tag{3}$$

holds, where

$$\chi_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$