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ON THE SPATIO-TEMPORAL STRUCTURE IN THE STOCHASTIC DIFFUSIVE SI MODEL

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Abstract. *In this paper, a stochastic diffusive infectious model in the population consisting of susceptibles and infectives is proposed. We consider the proliferation with the strong Allee effect, which means that there exists the optimal population density maximizes the per capita proliferation rate and it becomes negative below a certain threshold population density. By numerical simulations, we show that spatio-temporal patterns of the infectious spreading process become the fractal structure like the Sierpinski gasket independently of the existence of the noise in some parameter range, and in the other parameter range, the patterns under the noise is very different from the ones under the no noise case. Besides, we discuss the interaction between the invasion speed of infectives, the Allee effect and mortality of infectives.*
Keywords: Stochastic endemic model, Allee effect, Invasion speed, Fractal structure, Traveling wave, Numerical simulations

1. **Introduction.** Recently, many researches on infectious diseases in populations by various kinds of models [1-8] have been done from the mathematical and biological aspects. A large number of analytical models in epidemiology have been proposed in the past, including SI (susceptible-infected), SIR (susceptible-infected-recovered) and SIRV (susceptible-infected-recovered-vaccinated) models. With help from differential equations, cellular automata, etc., such models have been studied. However, most of conventional models are deterministic, and even if the models are stochastic ones, which are non-diffusive. Since in ecosystem, some sort of random fluctuation is often caused by changes in the environment, a precise analysis of ecosystem requires a stochastic model. Besides, in a detailed analysis of the infectious disease spread, we need to take account of spatial movement of each population. For these reasons, we consider a stochastic modeling of the infectious disease spread under spatial movement of each population.

Models in epidemiology can be classified into two types: one is an epidemic model which describes rapid outbreak of infectious diseases in less than one year, and the other is an endemic model which is used in modeling of diseases with longer epidemic period. In this paper, we propose the stochastic endemic model with spatial movement in populations consisting of susceptibles and infectives.

Controlling infectious diseases has been one of the major issues in epidemiology [9]. Prediction of the speed of the infection is thought to be important in controlling infectious diseases [10]. Hence, by means of the proposed stochastic diffusive endemic model, we study the invasion speed of infectives and the influence of the random fluctuation on the spatio-temporal behaviors of susceptibles and infectives. The analysis of the invasion speed of infectives and spatio-temporal behaviors of populations has a strong relation with