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OPTIMIZATION OF OBSERVATIONS FOR LQG CONTROL SYSTEMS BY AN INFORMATION THEORETIC APPROACH

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Abstract. *In this paper, we are concerned with an optimal selection of the gain matrix of the noisy observation for stationary LQG stochastic control systems. By introducing an information theoretic criterion based on a generalized Water Filling Theorem, we obtain a set of the observation gains which maximize the mutual information between the system state and the observations. Then, in order to find the optimal gain matrix from this set of gains which produces the best performance of the optimal LQG regulator, we solve an optimization problem with respect to the variables which are components of an orthogonal matrix. For easy numerical calculations of the solution, the constrained problem is converted to unconstrained one by introducing an n-dimensional polar coordinates system.*

Keywords: Gaussian processes, Kalman filter, LQG regulator, Optimal control

1. **Introduction.** For the Kalman filter [1], i.e., the linear Gaussian (LG) optimal state estimator, a considerable number of studies have been devoted to optimization of the observations [2]-[11]. These works are reviewed and summarized in Takeuchi [10] and we do not repeat here.

On the other hand, the situation is somewhat different in the optimization problem of the observations for the stochastic LQG optimal controlled systems. It is clear that in the LQG optimal state regulator systems, the control performance is more important than the estimation errors of the Kalman filter. Therefore, the observations should be designed and optimized to achieve the best performance of the LQG regulator. Nevertheless, no work has ever been devoted to this type of optimization of the observations, as far as the authors' knowledge is concerned. It seems that there is a misunderstanding about the stochastic LQG optimal control problem that "it is sufficient for us to simply apply the result of the corresponding deterministic optimal control problem". Of course, for the stochastic LQG optimal control problem, we have

(a) When the complete values of the state variables are available, the optimal control is the state feedback with the same feedback gain as the solution of the corresponding deterministic optimal control problem.
(b) Due to the separation theorem [12], even in the case when we have noisy observations, the optimal control remains in the form given by (a), except for the point that the state variables are replaced by the least-squares estimates of them.

However, as we will see, the performance of the LQG optimal regulator does depend on the selection of the gain matrix in the linear observation.

In this paper, we consider the optimization of observations for the LQG optimal controlled systems. For systems without the control input, we have already considered the