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PARAMETER ESTIMATION OF TERM STRUCTURES MODELED BY STOCHASTIC HYPERBOLIC SYSTEMS

Shin Ichi Aihara[1] and Arunabha Bagchi[2]

[1]Tokyo University of Science, Suwa
5000-1 Toyohira, Chino, Nagano, Japan
aihara@rs.suwa.tus.ac.jp

[2]FELab and Department of Applied Mathematics
University of Twente
Enschede, The Netherlands
a.bagchi@ewi.utwente.nl

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Abstract. *We consider a slight perturbation of the Hull-White short rate model and result in modified forward rate equation as studied in* [4]. *We develop the general framework of the Kalman filter for the stochastic hyperbolic system for the factor process of bonds. The noise covariance parameters included in the systems are estimated by calculating the quadratic variation of observation data. The market price of risk parameters are then identified with the aid of the maximum likelihood method.*

Key words: Interest rate models, Term structure, Forward curves, Stochastic hyperbolic equation, Kalman filter, MLE

1. **Introduction.** The modeling and parameter identification of term structures have been studied for the hyperbolic models [1, 13], parabolic models [7, 2, 5] and affine models [10]. Recently we proposed a new affine model, starting from the Hull-White model in [4]. We set the short rate model as

$$dr(t) = \{\Theta(t) - ar(t)\}dt + \sigma_r dW_r(t) \tag{1}$$

where W_r is a standard Brownian motion process. The bond price $P(t,T), 0 < T < \hat{T}$ is then given by

$$P(t,T) = \exp\{-\int_0^{T-t} [A(t,x) + B(t,x)r(t) + \int_0^t \sigma dw(s, x+t-s)]dx\} \tag{2}$$

where $w(t,x)$ denotes a two parameter Brownian motion process for presenting the small perturbation from the short rate model. The existence of the $w(t,x)$ process leads the one to obtain the MLE for the systems parameters through Kalman filter without adding the artificial observation noise as used in [8].

Here, we describe the conceptual difference between this paper and the other existing papers in [8, 14]. Usually without existence of $w(t,x)$ process, we can derive the hyperbolic system similar to (3), which is also an arbitrage free model. The corresponding observation equation is also possible to derive. To identify the system parameters, we need to get the exact form of likelihood functional through the Kalman filter. However, without w-noise, the noise covariance of the observation (12)(below) becomes singular. Hence we need to add the artificial noise to the observation data for estimating the parameters. This destroys the not arbitrage free property of our model, even though at first we make an arbitrage free model. Our new model (5)(below) is also an arbitrage free model and the