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ESTIMATION OF DENSITIES FOR HESTON-TYPE MODELS THROUGH THE MALLIAVIN-THALMAIER METHOD AND ITS APPLICATION TO THE CALCULATION OF GREEKS

Arturo Kohatsu-Higa[1] and Kazuhiro Yasuda[2]

[1]Graduate School of Engineering Sciences
Osaka University and Japan Science and Technology Agency
1-3, Machikaneyama, Toyonaka, Osaka, 560-8531, Japan
kohatsu@sigmath.es.osaka-u.ac.jp

[2]Faculty of Science and Engineering
Hosei University
3-7-2, Kajino-cho, Koganei-shi, Tokyo, 184-8584, Japan
k_yasuda@hosei.ac.jp

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Abstract. *In this paper, we estimate the joint density function of the stock price and its volatility for the Heston model and the double volatility Heston model through the Malliavin-Thalmaier formula. First, we give the representation of the joint density of the stock and its volatility. Next, we simulate these formulas on computer and compare them to other existing methods. We conclude from these experiments that our method has the smallest variance. Finally, we apply the formula to the calculation of Greeks in finance.*
Keywords: Density estimation, Malliavin-Thalmaier formula, Greeks

1. Introduction.

Financial systems require a careful continuous risk control in order to avoid undesirable results due to sudden large portfolio movements. One of the components in this problem is the risk control of option contracts.

From the mathematical point of view, the quantity that expresses this risk is the derivative of the price of the option price with respect to the parameters in the problem (these quantities are called "the Greeks" as many of them are denoted used capital Greek letters). In systems theory, this problem falls in the area of sensitivity analysis.

In this article, we deal with such a problem for binary options which are certain particular type of option contracts for which is difficult to compute these sensitivity quantities. In a particular case, this problem is equivalent to the estimation of density functions for random variables arising from solutions of stochastic differential equations. For a detailed description, see Section 4.

In general, such differential equations are not explicitly solvable and therefore the need for Monte Carlo simulation arises. The theory in order to estimate density functions through the Malliavin calculus has been available since the eighties. See Nualart [12] (Proposition 2.1.5) or Sanz-Solé [13] (Proposition 5.4). These formulae were applied in finance in Fournié et. al. [5] for one dimensional financial models.

When this formula is used to simulate multidimensional density functions one finds a "curse of dimensionality" problem. In fact, the classical formulae that give simulatable expressions for the density involve multiple iterated Skorohod integrals. The Skorohod