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ON THE SIMULATION ANALYSIS FOR THE STOCHASTIC INFECTIOUS MODEL WITH VACCINATION

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Abstract. *A mathematical model which describes the spread of the infectious disease has a very important role in the control, the prediction and the analysis of the infectious disease. In this paper, we study the modeling of the infectious disease in populations consisting of four populations: susceptible, infective, recovered and vaccinated ones. Noting that the actual infectious disease contains some kinds of random fluctuations caused by environmental changes, we propose the stochastic infectious model. By using the proposed stochastic model, we analyze the influence of the random fluctuation and vaccination on the spread of the infectious disease by numerical simulations.*
Keywords: Stochastic infectious model, Vaccination, Vaccination reproduction number, Backward bifurcation, Numerical simulations

1. **Introduction.** Recently, there has been a growing interest in controlling infectious disease [1-9]. One of public health strategies to control the infectious disease is vaccination. Hence, in this paper, we consider the infectious disease model in populations taking into consideration vaccinated population. Noting that in the actual infectious disease, environmental changes cause some kinds of random fluctuations in the spread of the infection, we propose the stochastic infectious model with vaccination. We study the influence of the random fluctuation and vaccination on the spread of the infectious disease by numerical simulations. It is well known that in the population without vaccination, if the basic reproduction number R_0 (the number of secondary cases that a single infected case will cause in the population) is less than one, then the infectious disease cannot invade the population. However, in the population with vaccination, even if the value of R_0 is less than one [1,10,11], the infectious disease successfully invades the population in some cases. This fact has a strong relation with the stability of the steady state of the infectious model, so we consider the stability of the steady state of the proposed model and study the relation between the stability and the spread of the infectious disease.

First, we begin with an explanation of the conventional infectious model [10] with vaccination. Consider the populations consisting of four classes: susceptible, infective, recovered and vaccinated, with the number at time t in each population denoted by $S(t), I(t), R(t)$ and $V(t)$ respectively. Interaction between each population is shown in Figure 1 [10]. In Figure 1, β presents the transmission rate, μ the death rate (= the birth rate), γ the recovery rate, ν the rate of loss of immunity, ϕ the vaccination rate, θ the rate of vaccination waning. A vaccination efficacy is denoted by $1 - \sigma$ with $\sigma \in [0, 1]$. In the case $\sigma = 1$, the vaccination is totally useless, and if $\sigma = 0$, vaccine is perfect.