International Journal of Innovative
Computing, Information and Control
Volume **7**, Number **1**, January **2011**

 ISSN 1349-4198
pp. **85–99**

# OPTIMIZATION OF LINEAR OBSERVATIONS FOR THE STATIONARY KALMAN FILTER UNDER A QUADRATIC PERFORMANCE CRITERION

Yoshiki Takeuchi

Dept of Information Science
Osaka University of Education
4-698-1 Asahigaoka, Kashiwara City, Osaka 582-8582, Japan
takeuti@cc.osaka-kyoiku.ac.jp

Received Feburary 2010; revised July 2010

Abstract. *In this paper, we are concerned with a problem of optimization of the linear observations which are used in the stationary Kalman filter. Especially, we consider the optimization of the gain matrix in the observation. In the previous works of the author, we introduced an information theoretic criterion based on a generalized Water Filling Theorem to obtain a gain matrix which minimizes the stationary error variance. The merit of this approach is that both analytical and numerical solutions are rather easily obtained compared with the case of the performance criterion which is quadratic in the estimation error and the gain matrix. In this solution process, however, the Riccati equation of the error covariance matrix reduces to a quasi linear equation, and the condition for the existence of the solution of this equation is somewhat stronger than that of the usual Riccati equation. This paper is concerned with the case of the quadratic performance criterion. By introducing an eigenvalue-eigenvector representation of a symmetric matrix which is a quadratic function of the gain matrix, we obtain an expression of the condition of optimality. Also, the numerical calculation of the solution is easily done by employing a multi-dimensional polar coordinates system. The results of numerical experiments show the efficiency of the algorithm.*
**Keywords:** Gaussian processes, Kalman filter, Least-squares estimation, Optimization of observation

1. **Introduction.** The optimization of observations in the least-squares estimation is one of classical problems in the theory of stochastic systems. However, after the pioneering studies in 1970's [1]-[4], this problem was not always popular because of mathematical difficulties, i.e., the nonlinearity in the error covariance equation. This nonlinearity exists even when the system and the observation are linear and Gaussian, i.e., even when we are concerned with the Kalman filter.

In the type of problem which is called the sensor allocation problem [10], the criterion is usually taken to be a quadratic function of the estimation error and the observation gain matrix. The condition of optimality for the observation gain was obtained in a form which cannot be easily solved by both analytical and numerical methods. Recently, we proposed a method of solution of this problem by introducing an information theoretic criterion [8]-[12]. Especially, for the discrete-time case, we have obtained an easily computable method of optimization for both stationary and nonstationary cases [10], [11] based on a generalized Water Filling Theorem [10].

In this paper, we consider the optimization of observations for the stationary Kalman filter under a performance criterion which is quadratic in the estimation error and the gain matrix. Our approach here, however, is different from that of the pioneering studies