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ON A TYPE I ERROR OF A RANDOM WALK HYPOTHESIS ON INTEREST RATES

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Abstract. *In the present paper, we will establish a law of large numbers for the sample covariance matrix of the forward rates when random walk hypothesis (RWH) on the spot rates holds. The study is motivated by the result in [10] that says the forward rates have much more factors than the stylized belief, and aimed to explain the result as a "type I error" on RWH. Our result in this paper shows that the number of factors of the forward rates is greater than that of the spot rates but at most twice.*

Keywords: Term structure of interest rates, Principal component analysis, Random walk hypothesis

1. **Introduction.** Management of risks is a most important issue in the modern financial industry. In the present days, the task is done relying on the asset pricing model based on no-arbitrage hypothesis combined with some statistical technics (see e.g., [5, 7, 8, 11, 12]). In the context of management of interest rate risks, principal component analysis is commonly used to detect "factors". In many studies (e.g., Litterman and Scheinkman [9], Buhler and Zimmermann [4], etc.), it is reported that the number of the factors is two – four, and this observation is reconfirmed in [10]: Table 1, Figures 1 and 2 below are taken from [10]. Similar results are commonly observed when applying PCA to spot interest rates, and the first factor is often called "parallel shift", the second one "twist" and the third one "butterfly move" or something similar.

In most cases, the analysis is implicitly performed by using a random walk hypothesis (henceforth RWH) on the spot rate; the increments of *spot* interest rates are independent and stationary (see Section 2.1 below). In the arbitrage-free framework in mathematical finance, however, it is not *spot*, but the *forward* rates that should be a random walk.

Recall that the drift condition instantaneous forward rate of Heath, Jarrow and Morton [6] is expressed as follows:

$$dr(t,x) = \sigma_r(t,T)\cdot dW_t^* = \sigma_r(t,x)\cdot dW_t + \sigma_r(t,x)\cdot\left(\int_0^x \sigma_r(t,y)dy + \lambda_t + \frac{\partial}{\partial x}r_t(x)\right)dt, \tag{1}$$

where W is a d-dimensional Brownian motion, $\sigma_r(\cdot,x)$ is an $\mathbf{R}^r$-valued process adapted to the natural filtration of W and λ is an adapted process. Here $r(t,x)$ is the instantaneous forward at time t that starts (and ends simultaneously) at $t+x$. One may see that r_t is a process with stationary independent increments only when $\frac{\partial}{\partial x}r_t(x) = 0$, which in turn means the interest rate moves only parallel shifts, and the number of the "factor" is always one.