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ON STABILITY OF THE VALUES OF RANDOM ZERO-SUM GAMES

Akio Tanikawa

Faculty of Information Science and Technology
Osaka Institute of Technology
Kitayama, Hirakata-shi, 573-0196, Japan
tanikawa@is.oit.ac.jp

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Abstract. *We consider a finite zero-sum two-person game determined by the payoff a_{ij} (to the first player P_1 from the second player P_2) corresponding to the strategies i and j ($1 \le i \le m, 1 \le j \le n$) selected respectively by P_1 and P_2. In particular, we deal with the case where each element a_{ij} is a random variable and discuss stability of the values $\{v_{mn}\}$ of the game as $m \to \infty$ and $n \to \infty$. The main stability result is derived from the law of large numbers for random linear programming problems. An upper bound and a lower bound of the game values are also obtained in this paper .*

Keywords: Zero-sum game, Matrix game, Random matrix, Game value

1. **Introduction.** We consider a finite zero-sum two-person game and denote two players respectively by P_1 (the first player) and P_2 (the second player). Each finite zero-sum two-person game is determined by the payoff a_{ij} (from P_2 to P_1) corresponding to the strategies i and j ($1 \le i \le m, 1 \le j \le n$) chosen respectively by P_1 and P_2. Thus, each game is represented by the payoff matrix A

$$A = (a_{ij}) = \begin{pmatrix} a_{11} & a_{12} & \cdots & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & \cdots & a_{mn} \end{pmatrix}$$

whose (i, j) element a_{ij} denotes the amount paid to P_1 by P_2 when P_1 selects the i-th row ($1 \le i \le m$) and P_2 selects the j-th column ($1 \le j \le n$). We denote the zero-sum game determined by the payoff matrix A simply by $G(A)$. Zero-sum two-person games are also called matrix games. We define the value v_{mn} of $G(A)$ by

$$v_{mn} = \max_{\boldsymbol{p}} \min_{\boldsymbol{q}} \boldsymbol{p}^T A\, \boldsymbol{q} = \min_{\boldsymbol{q}} \max_{\boldsymbol{p}} \boldsymbol{p}^T A\, \boldsymbol{q}\,, \tag{1}$$

where $\boldsymbol{p} = (p_1, p_2, \cdots, p_m)^T$, $p_i \ge 0$, $p_1 + p_2 + \cdots + p_m = 1$, $\boldsymbol{q} = (q_1, q_2, \cdots, q_n)^T$, $q_j \ge 0$, $q_1 + q_2 + \cdots + q_n = 1$ (see [25] and [29]). Here, T denotes transpose of a vector. In this paper, we consider the case where the elements a_{ij} of A are random variables, i.e., random matrix games, and discuss stability of the values $\{v_{mn}\}$ of the games when $m \to \infty$ and $n \to \infty$.

Although many research papers are concerned with pure Nash equilibria (i.e., saddle points) (see e.g., [5], [6], [15], [16], [18], [21], [22] and [24]), some papers are on the values of random matrix games (see [1], [3], [27] and [28]). More recent results on games can be found in [7], [8], [9] and [10].

Stability of the values of random matrix games was discussed firstly by Kabe (in [12]). He derived stability of random matrix games by applying the law of large numbers for