AN APPLICATION OF MERTON'S PORTFOLIO SELECTION PROBLEM TO THE CASE WITH A ZERO RECOVERY BOND

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ABSTRACT. *In an earlier paper Liang [6], the author studied an optimization problem of consumption and investment strategy for an investor who can invest in a riskless asset and a no-zero recovery defaultable bond. In this paper, we demonstrate some results of the case with a zero recovery bond, and these are the theoretical basics of Liang [6].*

Merton [7] was the first to formulate and solve such an optimization problem, and many researchers have extended and generalized the model of consumption and investment. However, optimization problems with defaultable assets have been rarely studied, though it is so important in today's financial market. This paper applies the framework of original Merton's model to a new market model that consists of a defaultable asset as well as a riskless asset. Under the assumption that the defaultable asset's price is modeled as a geometric Brownian motion with an unpredictable jump to zero, the optimal problem is reformulated and analytically solved.
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1. **Introduction.** The continuous-time portfolio problem has its origin in the pioneering work of Merton [7, 8] which deals with finding the optimal investment strategy of an investor. Many variations of the problem have been explored. For example, an utility function other than CRRA can be used; transaction costs can be introduced; the assumption of constant investment opportunities can be relaxed; additional assets can be added, for example, individual stocks; bankruptcy can be incorporated; etc. However, optimization problems with defaultable assets have been rarely studied, though it is so serious in today's financial markets.

Merton [8] considered a portfolio problem with a defaultable bond, but he used a bond model which can be seen as a rudimentary reduced form model with deterministic interest rates. Korn and Kraft [5] solved portfolio problems where investors can put funds into defaultable assets such as corporate bonds. To model default risks they used a firm value approach, which goes back to Black and Scholes [2] and Merton [9]. Bielecki and Jang [1] derived a solution for a representative investor who optimally allocates his wealth among the securities containing a defaultable bond. A recovery value is considered in the event of a default. Moore and Young [10] incorporate an insurable loss and random horizon into the classical framework of Merton. However they have not considered default risk, it is the key difference from this paper.

There are some most recently published papers about the portfolio selection problem, such as Chang and Chen [3], Wang et al. [12], Hasuike and Ishii [4] and Zhang and Li [13]. These papers will be introduced in next paragraph. The aim is to help place our contribution in context for readers; and to help further promote this work in wider communities.