

INVERSE PROBLEM FOR ELECTROMAGNETIC PROPAGATION IN A DIELECTRIC MEDIUM USING MARKOV CHAIN MONTE CARLO METHOD

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ABSTRACT. *This paper is concerned with a stochastic inverse methodology arising in electromagnetic imaging. Nondestructive testing using guided microwaves covers a wide range of industrial applications including early detection of anomalies in conducting materials. The focus of this paper is the identification of electromagnetic material parameters and emphasis is on one dimensional scattering of a dielectric slab. The direct problem can be solved numerically using the finite-difference time-domain method (FDTD). The Markov Chain Monte Carlo method (MCMC) is applied to the inversion problem. Some successful results of computational experiments are demonstrated in order to show the feasibility and applicability of the proposed method.*

Keywords: Microwave, Dielectric loss, Electrical cable, FDTD, Metropolis-Hasting algorithm

1. Introduction. Recently, interest has been growing in structural health monitoring (SHM) related to aging management of large scale systems, such as airplanes, bridges and nuclear power plants. Various kinds of nondestructive testing (NDT) techniques such as ultrasonic, eddy current, thermal are applied to the detection and the characterization of material damage. Combining NDT with simulation is a key component of future structural monitoring technologies. Mathematical descriptions of non-destructive testing (NDT) can be formulated as either a forward or an inverse problem for the physical domain of the inspection. A forward problem represents a real NDT system mathematically using the input and the output relationship with the appropriate admissible class of material parameters, while an inverse problem involves the construction of a method for recovering and/or visualizing material flaw information using the mathematical formulation of the forward problem. Figure 1 demonstrates the overall configuration of the proposed system. In this paper, a stochastic inverse methodology for NDT arising in electromagnetic imaging is investigated. Nondestructive testing using guided microwaves is used in a wide range of industrial applications including early detection of anomalies in conducting materials, analysis of human muscle tissue, etc. The characterization of material corrosion damage continues to be a very challenging problem. The problem is further complicated by the dispersive nature of the insulation covering the corrosion. Estimating the material properties in conventional measurements has been studied extensively [1]. For instance, L. F. Chen et al. have addressed the practical guidance on the development

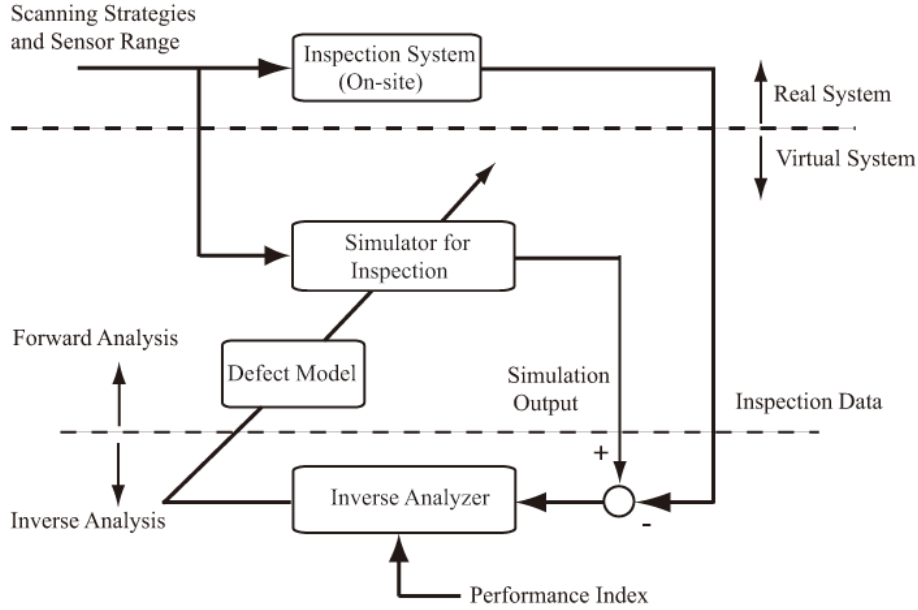


FIGURE 1. Diagnosis system using forward and inverse analysis

of suitable measurement methodologies using micro guided wave for a variety of materials in their book [1]. Our focus in this paper is in the identification of electromagnetic material parameters and the emphasis is on one dimensional scattering of a dielectric slab. Although prior work exists using nonlinear least square methods [2, 3, 4, 5, 6], it is well known that the problem mentioned above has many solutions due to the fact that it is ill-posed. Recently, interest has grown in stochastic inversion using Markov Chain Monte Carlo (MCMC) methods [7, 8]. Although the book written by L. F. Chen [1] et al. contains around thousands of references in a variety of applications, there is not any literature using MCMC to characterize the material properties. The author first proposed the parameter estimation method using the Gibbs sampling algorithm in [9]. The method has great advantages for the more practical aspects of inversions such as setting initial guesses and overcoming local minimums of the inverse solution. In this paper, a stochastic inversion technique based on the Metropolis-Hasting algorithm is applied to our problem.

2. Formulation of the Inverse Problem. Let $D_x(t, z)$, $H_y(t, z)$ and $E_x(t, z)$ be the component of the electric flux density, and the magnetic and electric field at time $t \in [0, T]$ and at location $z \in [0, Z]$. Using normalized electric flux densities and electric fields

$$\tilde{D}_x = \frac{1}{\sqrt{\epsilon_0 \mu_0}} D_x, \quad \tilde{E}_x = \sqrt{\frac{\epsilon_0}{\mu_0}} E_x, \quad (1)$$

and from Maxwell equations, the electromagnetic propagation in z -direction is governed by

$$\frac{\partial \tilde{D}_x(t, z)}{\partial t} = -\frac{1}{\epsilon_r \sqrt{\epsilon_0 \mu_0}} \frac{\partial H_y(t, z)}{\partial z} + J_s(t, z) \quad \text{in } [0, T] \times [0, Z] \quad (2)$$

$$\tilde{D}_x(\omega, z) = \epsilon_r^*(\omega) \tilde{E}_x(\omega, z) \quad (3)$$

$$\frac{\partial H_y(t, z)}{\partial t} = -\frac{1}{\sqrt{\epsilon_0 \mu_0}} \frac{\partial \tilde{E}_x(t, z)}{\partial z} \quad \text{in } [0, T] \times [0, Z] \quad (4)$$

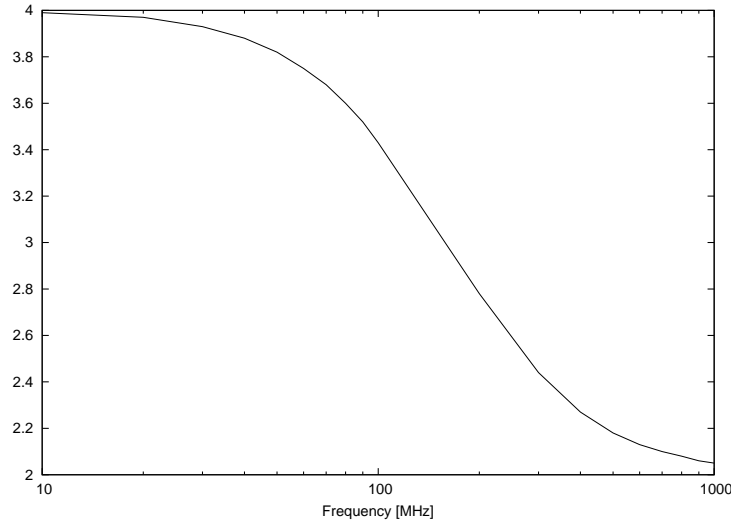


FIGURE 2. Dielectric constant for a Debye medium over the frequency range of 10 to 1000[MHz]

with zero initial states and absorbing boundary conditions. In system (2) and (4), ϵ_0 and μ_0 denote the dielectric constant and magnetic permeability of vacuum, respectively.

The source current J_s is the test signal in the inspection process and is given by

$$J_s(t, z) = \delta(z - z_s)g_s(t)I_{(0,t_s)}(t) \tag{5}$$

where δ denote the Dirac distribution and $z_s \in [0, Z_1)$ denotes the source point of test signal. The test signal is truncated at a finite time t_s by the indicator function I .

The spatial domain $[0, Z]$ is decomposed into two separate regions. The region $z \in [0, Z_1)$ is air and is assumed to have zero electric polarization and zero conductivity. Hence, in (3), it becomes

$$\epsilon_r^* = 1. \tag{6}$$

A target material in $z \in [Z_1, Z]$ is assumed to be a chemical material such as a polymer where the dielectric constant and the conductivity are dispersive. A material like this is well approximated by the Debye law described by

$$\epsilon^*(\omega) = \epsilon_r + \frac{\sigma}{j\omega\epsilon_0} + \frac{\chi_1}{1 + j\omega\tau} \tag{7}$$

where ϵ_r , σ , χ_1 and τ denote a dielectric constant, conductivity, and frequency-dependent parameters [3]. As indicated in Equation (7), a target material includes a medium whose dielectric constant and conductivity vary over the frequency range. Figures 2 and 3 show a relative dielectric constant and conductivity as functions of frequency for a Debye medium (7) with properties (21). From (7), it follows that the dielectric constant for a Debye medium can be represented as

$$\epsilon_r + \frac{\chi_1}{1 + (\tau\omega)^2}$$

and the frequency dependent conductivity can be rewritten as

$$\omega\epsilon_0 \left\{ \frac{\sigma}{\epsilon_0\omega} + \frac{\chi_1\omega\tau}{1 + (\tau\omega)^2} \right\},$$

respectively.

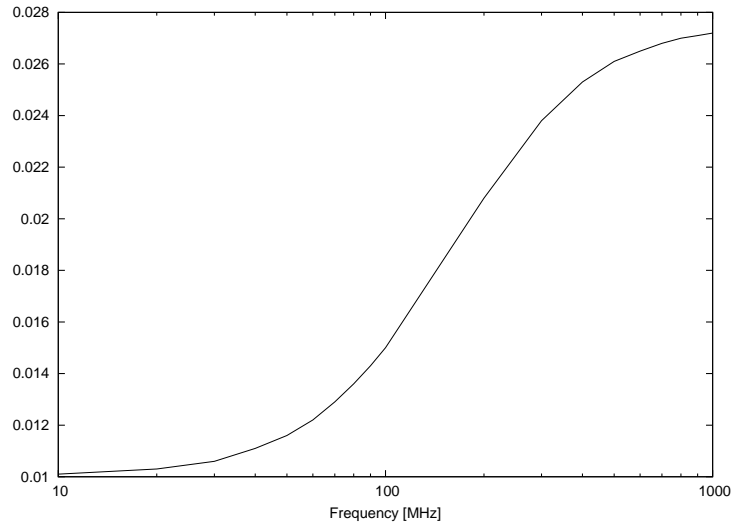


FIGURE 3. Conductivity for a Debye medium over the frequency range of 10 to 1000[MHz]

There are many interesting applications for identifying frequency dependent parameters in the Debye model (7). Thus our inverse problem is to identify dielectric parameters

$$\mathbf{q} = \{\epsilon_r, \sigma, \chi_1, \tau\}$$

from observations near the frequency dependent medium:

$$Y(t; \mathbf{q}) = E_x(t, 0; J_s) \quad (8)$$

where $E_x(t, z)$ is the solution of (2)-(4).

3. Numerical Scheme of Direct Problem. The detection signal (8) corresponding to (5) can be obtained by numerical simulation using the finite-difference time-domain (FDTD) method [10]. FDTD uses central-difference approximations to the space and time partial derivatives. The resulting finite-difference equations are solved in a so-called “leapfrog manner”. More precisely, the electrical field components in a volume of space are solved at a given instant in time. Then the magnetic field components in the same spatial volume are solved at the next instant in time.

Taking the central difference formula for both the temporal and spatial derivatives, Equations (2) and (4) can be approximated by

$$\begin{aligned} & \frac{\tilde{D}_x^{n+1/2}[k] - \tilde{D}_x^{n-1/2}[k]}{\Delta t} \\ &= -\frac{1}{\epsilon_r \sqrt{\epsilon_0 \mu_0}} \frac{H_y^n[k + 1/2] - H_y^n[k - 1/2]}{\Delta z} + J_s^n[k] \end{aligned} \quad (9)$$

$$\begin{aligned} & \frac{H_y^{n+1}[k + 1/2] - H_y^n[k + 1/2]}{\Delta t} \\ &= -\frac{1}{\sqrt{\epsilon_0 \mu_0}} \frac{\tilde{E}_x^{n+1/2}[k + 1] - \tilde{E}_x^{n+1/2}[k]}{\Delta z} \end{aligned} \quad (10)$$

where n implies time division, i.e., $t = \Delta t \cdot n$ and where the terms in parentheses represent grid point in the spatial domain, i.e., $z = \Delta z \cdot k$, respectively. In a one dimensional spatial domain, there exist two boundary points $z = 0, Z$. An absorption boundary condition is necessary at both end points, $z = 0, Z$ in order to keep the outgoing electrical and

magnetic fields E and H from being reflected back into the target domain. This can be accomplished by setting

$$\tilde{D}_x^n[0] = \tilde{D}_x^{n-1}[1] \tag{11}$$

$$\tilde{E}_x^n[0] = \tilde{E}_x^{n-1}[1] \tag{12}$$

$$\tilde{D}_x^n[K] = \tilde{D}_x^{n-1}[K - 1] \tag{13}$$

$$\tilde{E}_x^n[K] = \tilde{E}_x^{n-1}[K - 1] \tag{14}$$

where $Z = K \cdot \Delta z$. To transform Equation (3) into a time domain difference equation for implementation into FDTD, the frequency term should be replaced by the time domain representation. Noting that $1/j\omega$ in the frequency domain is equivalent to integration in the time domain, Equation (3) becomes

$$\tilde{D}_x(t) = \epsilon_r \tilde{E}_x(t) + \frac{\sigma}{\epsilon_0} \int_0^t \tilde{E}_x(t') dt' + \frac{\chi_1}{\tau} \int_0^t \exp\left(-\frac{t' - t}{\tau}\right) \tilde{E}_x(t') dt'. \tag{15}$$

By approximating Equation (15) as a summation in the sampled time domain, the numerical scheme can be represented by

$$\tilde{D}_x^n = \epsilon_r \cdot \tilde{E}_n + I_n + S_n \tag{16}$$

where

$$I^n = I^{n-1} + \frac{\sigma \cdot \Delta t}{\epsilon_0} \cdot \tilde{E}_x^n \tag{17}$$

$$S^n = \exp\left(-\frac{\Delta t}{\tau}\right) S^{n-1} + \chi_1 \cdot \frac{\Delta t}{\tau} \cdot \tilde{E}_x^n \tag{18}$$

starting with zero initial states with respect to both \tilde{D}_x^0 and \tilde{E}_x^0 . Consequently, the forward problem can be implemented by the repeated scheme mentioned above:

$$Y^n(\mathbf{q}) = \sqrt{\frac{\mu_0}{\epsilon_0}} \tilde{E}_x^n[0](J_s). \tag{19}$$

Figure 4 depicts a simulation result for a direct problem (19). A sinusoidal wave was applied to the test signal at $z_s = 5 \cdot \Delta z$ in Equation (5):

$$g_s^n = \sin(2 \cdot \pi \cdot f \cdot \Delta t \cdot n) \tag{20}$$

where f determines the frequency of the test signal. In the experiment, a sinusoidal wave of $700MHz$ is applied to a dielectric medium with parameters

$$\epsilon_r = 2.0, \quad \sigma = 0.01, \quad \chi_1 = 2.0, \quad \tau = 0.001[\mu sec]. \tag{21}$$

4. Stochastic Inverse Methodology. The output least square method is a conventional inverse methodology which seeks the optimal solution of

$$J(\mathbf{q}) = \min_{\mathbf{q} \in Q} \sum_{i=1}^K |Y(t_i; \mathbf{q}) - Y_d^i|^2. \tag{22}$$

However, it is well known that those experimental tests might not achieve good convergence results because of the complicated dependence in the related direct problem (8). In this paper, we propose a new stochastic inverse methodology for identifying those parameters. Using a Bayesian formula, the full probability model is specified by the deterministic Formula (8). The posteriori density function with respect to the set of unknown parameter vector can be represented by

$$\pi(\mathbf{q}|\mathbf{Y}_d) \propto l(\mathbf{Y}_d|\mathbf{q})\pi(\mathbf{q}). \tag{23}$$

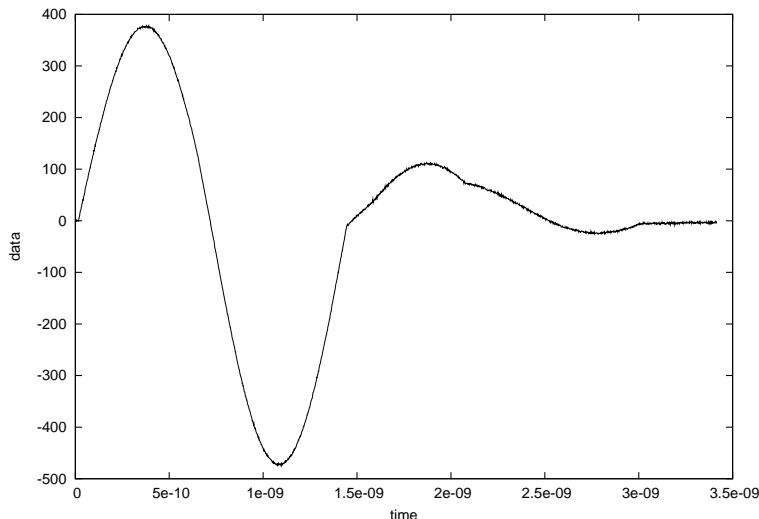


FIGURE 4. Simulation result

Suppose that measurements are made

$$Y_d^i = Y(t_i; \mathbf{q}) + \eta_i, \quad \eta_i \sim N(0, \xi^2), \quad (i = 1, 2, \dots, K). \tag{24}$$

Then the likelihood functional can be written by the following form:

$$l(\mathbf{Y}_d | \mathbf{q}) \propto \prod_{i=1}^K \frac{1}{\sqrt{2}\xi} \exp\left(-\frac{|Y(t_i; \mathbf{q}) - Y_d^i|^2}{2\xi^2}\right). \tag{25}$$

In our estimation algorithm, prior distributions for unknown parameters $\pi(\mathbf{q})$ are specified as

$$\pi(\mathbf{q}) = \prod_{l=1}^4 \pi(q_l). \tag{26}$$

Then we specify full conditional distributions for posteriori density function through the likelihood functionals. The full conditional distribution $\pi(q_l | \dots)$ ($l = 1, 2, 3, 4$) has the representation

$$\pi(q_l | q_{-l}, \mathbf{Y}_d) \propto \pi(q_l) \prod_{i=1}^K \frac{1}{\sqrt{2}\xi} \exp\left(-\frac{|Y(t_i; \mathbf{q}) - Y_d^i|^2}{2\xi^2}\right) \tag{27}$$

where q_{-l} denotes the remaining component except q_l .

Our estimation algorithm is based on sampling procedures from the posteriori distribution from which a sample can be drawn via Markov chains. To this end, a transition kernel $p(q, \phi)$ must be constructed in such a way that the posteriori function Equation (23) is the equilibrium distribution of the chain. A simple way to implement this is to consider the reversible chains where the kernel p satisfies

$$\pi(q)p(q, \phi) = \pi(\phi)p(\phi, q). \tag{28}$$

The feature of the Metropolis-Hasting algorithm is that this reversible kernel p is constructed by

$$p(q, A) = \int_A k(q, \phi)\alpha(q, \phi)d\phi + I(q \in A) \left\{ 1 - \int k(q, \phi)\alpha(q, \phi) \right\} d\phi \tag{29}$$

TABLE 1. Estimation summary using MH algorithm

Quantities	True Values	Estimated Values
ϵ_r	2.00	1.99
$\log_{10} \sigma$	-2.00	-1.95
χ_1	2.00	1.68
$\log_{10} \tau$	-9.00	-9.05

for any subset A of the parameter space. In the above expression, the acceptance probability

$$\alpha(q, \phi) = \min \left\{ 1, \frac{\pi(\phi)k(\phi, q)}{\pi(q)k(q, \phi)} \right\} \tag{30}$$

plays an essential role in MH algorithms [8]. There exist varieties of the proposal kernel functions k for practical implementations. Thus the process version of the MH algorithm is given by the following steps:

Step 1: Initialize the iteration counter $j = 1$ and set the initial guess of the chain \mathbf{q}^0 .

Step 2: Initialize the component $i = 1$.

Step 3: Move the i th component of the parameter vector of the chain to a new value ϕ_i generated from the prescribed transition kernel $k_i(q_i^{(j-1)}, \phi_i)$.

Step 4: Calculate the acceptance probability of the move $\alpha_i(q_i^{(j-1)}, \phi_i)$ given by Equation (30). If the move is accepted, update the chain $q_i^{(j)} = \phi_i$. If the move is rejected, set $q_i^{(j)} = q_i^{(j-1)}$.

Step 5: Change the counter from i to $i + 1$ and return to Step 3 until the dimension of parameter vector, i.e., $dim(\mathbf{q}) = 4$.

Step 6: Change the counter from j to $j + 1$ and return to Step 2 until convergence is reached.

5. Simulation Experiments. Throughout our numerical experiments, simulation data are generated using the direct problem in Section 3. Taking into account that actual measurement procedures can be well approximated by Equation (24), some random noise is added to the simulation data to test our inverse methodology. The second variance term in Equation (24) was provided using a conventional Gaussian random generator. The set of true parameters in the experiments is selected by Equation (21). Table 1 shows the estimated results in the experiments.

6. Conclusion. The inverse methodology arising in electromagnetic imaging was discussed within the Bayesian framework. We propose the statistical method for identifying electromagnetic material parameters on a one dimensional scattering problem of a dielectric slab. The direct problem was obtained by numerical simulation using FDTD method. The stochastic inverse problem was solved via the Metropolis-Hasting algorithm.

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